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ABSTRACT

Behavioral objectives for geometry, algebra, computer mathematics, trigonometry, analytic geometry, calculus, and probability are specified for grades 10 through 12. General objectives are stated for major areas under each topic and are followed by a list of specific objectives for that area. This work was prepared under an ESEA Title III contract. (DT)

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U.S. DEPARTMENT OF HEALTH
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

Secondary Schools

CURRICULUM

GUIDE

Cranston School Department
Cranston, Rhode Island
1972

MATHEMATICS

Grades 10-12 Levels 87-112

ED 077740

Secondary School
CURRICULUM GUIDE

DRAFT COPY

Prepared By

a curriculum writing team

of secondary teachers

Project PACESETTER

Title III, E. S. E. A., 1965

Cranston School Department

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PREFACE

The following levels consist of instructional objectives and activities for each course of study within every curriculum area. These materials were produced by a staff of teachers working on curriculum teams for Project PACESETTER. They are, therefore, the product of the experience of the professionals who will put them to use.

This curriculum guide provides each teacher with curriculum materials organized into behavioral objectives with a scope and sequence. The guide is intended to encourage feedback so that a fully classroom tested curriculum will eventually result from the participation and suggestions of all teachers in the secondary schools of Cranston.

OBJECTIVES IN TERMS OF LEARNING VARIABLES

Bloom and his collegues devised a taxonomy of educational objectives designed to classify the behavior of students in three domains as a result of having participated in a series of instructional experiences. The three domains are the cognitive (intellectual), the affective (emotional), and the psychomotor (physical). Within each of these domains there is a hierarchy which denotes increasing complexity of learning which is shown below.

Cognitive	Affective	Psychomotor
knowledge	receiving	frequency
comprehension	responding	energy
application	valuing	duration
analysis	organization	
synthesis	characterization	
evaluation		

The objectives which appear in these Curriculum guides have been stated in behavioral or performance terms. In addition to the general technique of the behavioral statement, the authors were careful to differentiate the levels at which given behaviors could be expected of the student. Thus, in the cognitive domain, a student's performance in the display of knowledge of a concept is less complex than the student's performance when he applies the concept in a given situation. Similarly, in the affective domain, a response to a situation is not as complex as the display of a value toward a given situation.

In initial classroom trials of this curriculum teachers will evaluate the appropriateness of the objectives and make recommendations for revising, deleting, or adding to the objectives or activities.

LEVELS, OBJECTIVES, AND ACTIVITIES

The curriculum guides provided here are organized into behavioral objectives which generally include two major components. The first is the objective statement which specifies the behavioral variable--the intended behavior of the students as a result of having participated in a set of instructional experiences, the content or topic and the evaluative criterion which is sometimes implicit in the behavioral objective. Curriculum writers have made every effort to classify the intended behaviors in keeping with the work of Bloom and others. The objectives, then, are stated in terms of specific behaviors which range from the simple, such as memorizing or translating, to the most complex, such as synthesizing or evaluating. The second major component is comprised of activities which outline what the student should do to attain the objective. These activities are suggested and should be added to, deleted, or modified by the teacher according to the needs and characteristics of individual students and the teacher's own experience and knowledge.

It is important to note here that the objectives serve the purpose of helping each teacher select appropriate learning experiences, communicate to others what is expected, and provide both student and teacher with a standard for evaluating progress. Objectives should not be seen as limiting teacher innovation or what the student is expected to know.

Each of the curriculum areas is divided into major topics or "Levels." Each level begins with a level objective which is followed by numbered objectives subordinate to it. Suggested activities follow each of these specific objectives and are numbered consecutively throughout the level.

EVALUATIVE CRITERIA

Many of the evaluative statements included in the behavioral objectives are teacher oriented; final decisions on evaluation have traditionally been the prerogative of the teacher. As we move toward continuous progress and, eventually, individualized instruction, it is hoped that the evaluation component increasingly becomes the shared responsibility of both teacher and student.

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* pages are numbered within levels only'

LEVEL OBJECTIVE

THE STUDENT WILL DEMONSTRATE AN INCREASE IN HIS ABILITY TO APPLY THE GEOMETRIC CONCEPTS OF SETS, INDUCTION, DEDUCTION, AND ANGLE RELATIONSHIPS BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Sets and Set Relations

Objective #1: The student will increase in comprehension of the basic elements of sets and set relations by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Identify a well-defined set by the roster and descriptive methods.
2. Describe sets as being finite or infinite.
3. Distinguish between the empty set and the set containing the element 0.
4. Define subset and to differentiate and illustrate proper subsets, improper subsets, overlapping sets and disjoint sets.
5. Find the union, intersection, and complement of specified sets.
6. Use Venn diagrams to illustrate the operations among related sets.
7. Identify the set of real numbers and its subsets.
8. Explain the concept of one-to-one correspondence between the real numbers and points on a line.
9. Define absolute value and show its application regarding distance between points.
10. Explain the symbols used in comparing real numbers.

11. Recognize and interchange various forms of rational numbers.

II. Induction and Deduction

Objective #2: The student will increase his ability to think critically by applying principles of induction and deduction in the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

12. Explain the meaning, importance, and limitations of inductive thinking.

13. Apply induction to a problem situation in order to reach a conclusion.

14. Recognize intuition as a tool in gaining insight in problem situations.

15. Explain the meaning, importance, and limitations deductive thinking.

16. Apply deduction to a problem situation in order to reach a conclusion.

17. Recognize deduction as a system used in reaching conclusions.

18. Constrast among intuition, inductive and deductive thinking.

III. Nature of a Deductive System

Objective #3: The student will apply the basic concepts of geometry to formulating definitions, postulates, and theorems by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

19. Explain the statement that point, line, and plane are being described and not defined and they constitute the three basic undefined terms of a Euclidian geometric system.

20. Explain the statement that precise definitions would involve the introduction of less familiar terms which would in turn have to be defined.

21. Recognize a good definition.

22. Interpret postulates and theorems as they apply to a deductive system.

23. Demonstrate a knowledge of the basic terms of beginning logic by indicating correct conclusions, converses, contrapositives, inverses, etc.

24. Apply postulates and theorems concerning points, lines, and planes.

25. Apply the ideas which relate to the correspondence of the set of real numbers and the set of points on a line.

26. Apply the Betweenness of Points definition.

27. Recognize that a segment has exactly one mid-point.

IV. Angle Relationships

Objective #4: The student will apply definitions, postulates, and theorems to proofs involving angles and angle relationships by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

28. Apply the angle measurement postualte and the protractor postulate.

29. Explain that the protractor postulate does for angles and angle measures what the ruler postulate does for segments and lengths.

30. Apply the angle addition theorem which will be used extensively in proofs.

31. Apply theorems related to angles and angle relationships.

32. Apply those theorems and definitions related to perpendicular lines and right angles.

33. Apply those theorems and definitions related to complementary, supplementary, and vertical angles.

34. Write a complete demonstration of a two-column proof.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY THE DEFINITIONS AND THEOREMS OF PARALLEL LINES, PARALLEL PLANES, POLYGONS, AND CONGRUENCE TO DEVELOPING TECHNIQUES OF PROOF BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Parallel Lines and Planes

Objective #1: The student will apply definitions and theorems involved in working with parallel lines and planes and to develop techniques for proving examples involving them by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Apply definitions as reasons for steps in proofs and as generators of algebraic equations in order to solve number problems.

2. Recognize and identify by name all angle pairs formed by two lines intersected by a transversal.

3. Give measures of all angles formed when two parallel lines are cut by a transversal providing one of those angles is known.

4. Prove whether or not two lines are parallel when given the measures of various angles formed by two lines cut by a transversal.

5. Recognize when auxiliary lines are necessary to complete a proof and justify their use.

6. Realize that not all statements can be proven directly.

7. Recognize when a statement cannot be proven directly and prove the statement by means of an indirect proof.

II. Parallel Lines Applied to Polygons

Objective #2: The student will apply the concepts of parallel lines and planes to the study of polygons by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

8. Distinguish between polygons and other types of geometric figures as well as between various types of polygons.

9. Recognize and identify various types of triangles and quadrilaterals.

10. Apply parallel lines to proving various properties of triangles and quadrilaterals.

11. Apply properties of triangles and quadrilaterals to proofs.

12. Apply special properties of triangles and quadrilaterals to the solution of number problems.

13. Identify an exterior angle of a triangle and understand its relationship to the remote interior angles of the triangle.

14. Explain the properties of a polygon dealing with the sum of its interior and exterior angles.

15. Find the measure of an interior or exterior angle of a regular polygon.

III. Congruence

Objective #3: The student will apply the various cases of congruence to proofs involving congruent triangles by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

16. Recognize and name correspondences between two geometric figures.

17. Interpret the concept of congruence.

18. Apply all ways of proving triangles congruent: SAS, ASA, AAS, SSS and the additional methods for right triangles: HA, HL, LL, LA. (SSA and AAA are never cases of congruence)

19. Define congruence of triangles as an equivalence relation.

20. Recognize congruent triangles when they are overlapping and prove them congruent.

21. Apply congruent triangles to prove that segments are congruent and angles are congruent.

22. Apply congruent triangles and corresponding parts to demonstrate special properties of triangles.

23. Apply congruent triangles and corresponding parts to demonstrate properties of special triangles.

LEVEL OBJECTIVE

THE STUDENT WILL INCREASE IN COMPREHENSION OF THE STRUCTURE OF MATHEMATICAL SYSTEMS PARTICULARLY THE REAL NUMBER SYSTEM, THE ELEMENTS OF LOGIC, AND THE COMPUTATION WITH ALGEBRAIC EXPRESSIONS BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Real Number System

Objective #1: The student will increase in comprehension of the real number system by distinguishing among the various categories of the real number system by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Distinguish between types of numbers.
2. Interpret properties exhibited by particular sets of numbers.
3. Perform operations using particular sets of numbers.
4. Describe the structure of the Real Number System.
5. Distinguish between periodic and non-periodic decimals.
6. Express periodic decimals as the quotient of two integers.

II. Logic

Objective #2: The student will apply the basic elements of mathematical logic to given statements and proofs by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

7. Apply inductive reasoning.

8. Translate notation used in mathematical reasoning.

9. Apply deductive reasoning.

10. Distinguish between induction and deduction.

11. Form the negation of a statement.

12. Form basic composite statements (conjunction, disjunction, conditional, and biconditional).

13. Assign and organize in table form the truth values of composite statements.

14. Form negations of composite statements.

15. Form the variants of a conditional.

16. Explain the nature of a tautology.

17. Interpret the nature of a deductive system.

18. Apply a deductive system to proofs.

III. Mathematical Structure

Objective #3: The student will increase in comprehension of the structure of mathematical systems by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

19. Identify unary and binary operations.

20. Describe the concept of a group.

21. Identify properties exhibited by a group.

22. Describe the concept of a commutative (Abelian) group.

23. Identify properties exhibited by an Abelian group.

24. Describe the concept of a field.

25. Identify the properties exhibited by a field.

26. Recognize the set of real numbers as a field and specifically, as a complete ordered field.

IV. Algebraic Expressions

Objective #4: The student will apply the principles of algebraic expressions to computation within the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

27. Evaluate algebraic expressions.

28. Simplify expressions using grouping symbols.

29. State the order of algebraic operations.

30. Perform the operations of addition and subtraction using algebraic expressions.

31. Perform the operation of multiplication using the following algebraic expressions:

    a. monomial X monomial,

    b. monomial X binomial,

    c. binomial X binomial,

    d. monomial X multinomial, and

    e. multinomial X multinomial.

32. Perform the operation of division using the following algebraic expressions:

    a. multinomial monomial,

    b. Long Division, and

33. Factor algebraic expressions such as the following:
    a. common monomial factoring,
    b. common binomial factoring,
    c. difference between two perfect squares,
    d. perfect trinomial square,
    e. general trinomial, and
    f. $a^n \pm b^n$.
34. Perform the expansion of binomials.
35. Perform the square of a trinomial.
36. Simplify algebraic fractions.
37. Perform the addition and subtraction of algebraic fractions.
38. Perform the multiplication of algebraic fractions.
39. Perform the division of algebraic fractions.
40. Simplify complex fractions.
41. Perform operations involving positive integral exponents.
42. Perform operations involving negative integral exponents.
43. Perform operations involving fractional exponents.
44. Translate fractional expenents to radicals.
45. Express a radical in simplest form.
46. Perform operations with radicals.
47. Simplify expressions with fractional exponents.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY THE PROPERTIES OF RELATIONS AND FUNCTIONS AND METHODS OF SOLVING EQUATIONS TO A VARIETY OF PROBLEM SOLVING SITUATIONS BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Solutions of Open Sentences in One Variable

Objective #1: The student will increase in comprehension of problem solving involving open sentences in one variable by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Solve a first-degree equation in one variable.
2. Distinguish between a conditional equation and an identity equation.
3. Apply the operation laws for inequalities to the solution of first-degree inequalities in on variable.
4. Graph the solutions of open sentences on a number line.
5. Solve composite open sentences and graph their solutions.
6. Solve open sentences involving absolute value.
7. Solve quadratic equations by factoring.
8. Solve quadratic equations by completing the square.
9. Solve the general quadratic equation $ax^2 + bx + c = 0$ by completing the square.
10. Solve quadratic equations by using the Quadratic Formula.

11. Interpret the discriminant to determine the nature of the roots of a quadratic equation.

12. Write a quadratic equation from its given roots by the method of the sum and product of the roots.

13. Solve quadratic inequalities.

14. Solve irrational equations and identify extraneous roots.

15. Solve rational equations and identify extraneous roots.

16. Apply the techniques of solving linear and quadratic equations to problem solving.

17. Analyze and organize given information in a verbal problem in order to form an appropriate open sentence.

18. Solve different types of verbal problems such as: age, coin, consecutive integer, digit, geometry, mixture, motion, and work.

II. Relations and Functions

Objective #2: The student will increase in comprehension of the concepts of functions and relations by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

19. Define a relation.

20. Graph a relation on a Cartesian coordinate system.

21. Define a function.

22. Differentiate between a relation and a function.

23. Graph a function on a Cartesian coordinate system.

24. Form the inverse of relations and functions and compare their graphs.

25. Use functional notation.

26. Form and graph the sum, product, difference, quotient, and composition of functions.

III. Linear Functions

Objective #3: The student will apply the concepts of linear functions to equations and their graphs by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

27. Recognize the equation of a linear function.

28. Determine whether a line is vertical, horizontal, or oblique.

29. Define the slope of a line.

30. Write the equation of a straight line in:

    a. point-slope form,

    b. slope-intercept form, and

    c. standard form.

31. Recognize that the slopes of parallel lines are equal.

32. State the relationship between the slopes of perpendicular lines.

33. Apply the distance formula.

34. Apply the midpoint formula.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY GRAPHING AND OTHER EQUATION SOLVING TECHNIQUES INVOLVING CONIC SECTIONS, MORE THAN ONE VARIABLE, EXPONENTS, AND LOGARITHMS BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Conic Sections

Objective #1: The student will apply the properties of conic sections to graphing these sections and solving equations by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Understand the formation of the four sections of a right circular conical surface.
2. Identify the equations of a circle, parabola, ellipse, and hyperbola.
3. Graph the four conic sections at the origin.
4. Translate the four conic sections to the point (h, k).
5. Solve and graph linear-quadratic systems of equations.
6. Solve and graph quadratic-quadratic systems of equations.
7. Solve and graph systems of quadratic inequalities in two variables.
8. Apply quadratic systems to the solution of verbal problems.

II. Solutions of Equations in more than one Variable

Objective #2: The student will apply graphing and the properties of equations to the solution of various systems of equations by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

9. Classify and identify systems of two simultaneous equations in two variables.

10. Find solution sets of systems of simultaneous linear equations in two variables graphically.

11. Find solution sets of systems of simultaneous linear equations in two variables algebraically:

    a. by the multiplication-addition method, and

    b. by the subtraction method.

12. Find solution sets of the general system of two equations in two variables.

13. Solve algebraically systems of three equations in three variables.

14. Solve algebraically and graphically systems of one first-degree equation and one second-degree equation.

15. Solve algebraically and graphically systems of two second-degree equations.

16. Solve algebraically and graphically systems of inequalities.

III. Exponential and Logarithmic Functions

Objective #3: The student will apply the exponential and logarithmic functions to the solution of arithmetic and word problems by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

17. Distinguish between algebraic and transcendental numbers.

18. Recognize and graph an exponential function.

19. Define a logarithm.

20. Identify and graph a logarithmic function.

21. Recognize exponential and logarithmic functions as inverses.

22. Interchange exponential and logarithmic notation.

23. Apply exponential laws to properties of logarithms.

24. Apply the properties of logarithms to any base.

25. Solve equations involving logarithms to any base.

26. Explain the development of the common logarithm.

27. Use a table of mantissas to find the common logarithm of a real number.

28. Use a table of mantissas to find antilogarithms.

29. Use the method of linear interpolation to approximate mantissas and antilogarithms not found in a table.

30. Organize and use common logarithms to perform the operations of multiplication, division, raising to powers, and extracting roots.

31. Solve exponential equations using common logarithms.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY SUCH CONCEPTS AS COMPLEX NUMBERS, MATHEMATICAL INDUCTION, PROBABILITY, AND TRIGONOMETRY TO PROBLEM SOLVING SITUATIONS BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Complex Numbers

Objective #1: The student will apply the concept of complex numbers to problem solving by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Define a complex number.
2. Express any number in complex form.
3. State the sequence of the powers of "i."
4. Perform the operations of addition, subtraction, multiplication, and division of complex numbers.
5. Graph complex numbers on a coordinate plane.
6. List the properties of the complex field.

II. Sequences, Series, and Mathematical Induction

Objective #2: The student will apply the properties of probability to solving problems about uncertainity and chance by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

7. Identify an arithmetic sequence of numbers.
8. Find the nth term and the sum of the first n terms in an arithmetic series.

9. Identify a geometric sequence of numbers.

10. Find the nth term and the sum of the first n terms in a geometric series.

11. Solve verbal problems involving arithmetic and geometric sequences.

12. Insert arithmetic and geometric means between two extremes.

13. Solve verbal problems involving arithmetic and geometric series.

14. State the axiom of mathematical induction.

15. Use mathematical induction in proofs.

III. Permutations, Combinations, and Probability

Objective #3: The student will apply the properties of probability to solving problems about uncertainity and chance by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

16. Define a permutation.

17. Solve verbal problems involving permutations.

18. Define a combination or a selection.

19. Distinguish between a permutation and a combination type problem.

20. Solve verbal problems involving combinations.

21. Identify the basic probability situations.

22. Solve verbal problems involving probabilities.

IV. Trigonometry

Objective #4: The student will apply trigonometric functions to problem solving situations by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

23. Define an angle in standard position.

24. Define the six trigonometric functions.

25. Recognize the relationships that exist among the trigonometric functions.

26. Manipulate the fundamental identities.

27. Find the trigonometric functions of special angles.

28. Find the trigonometric functions of any angle using the tables of values.

29. Interpolate the values of the trigonometric functions.

30. Solve right triangles.

31. To solve right triangles in verbal problem situations.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY THE PRINCIPLES OF GEOMETRY BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Common Sense and Organized Knowledge

Objective #1: The student will increase in knowledge of the origin and definitions of geometry by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Trace the early histroy of Geometry from the Greeks and Egyptians.
2. Recognize the common sense used by the Egyptians.
3. Recognize the logical reasoning used by the Greeks.
4. Draw geometric abstractions.
5. Define a theorem as a statement which can be proved.
6. Define postulates and axioms as statements which are accepted without proof.
7. Accept point, line, and plane as the basic undefined terms in Euclidian Geometry.
8. Draw representations for the basic undefined terms.

II. Sets and Real Numbers

Objective #2: The student will increase in comprehension of sets and the real number set in particular by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

9. Define and use the following terminology:
    a. set,
    b. "is an element of,"
    c. subset,
    d. empty set,
    e. intersection of two sets, and
    f. union of two sets.
10. Construct the subsets of a given set.
11. Determine the intersection of two sets.
12. Determine the union of two sets.
13. Establish one-to-one correspondences between sets.
14. Recognize the structure of the real number system.
15. Use the notations and .
16. State the following basic properties of the real number system:
    a. Uniqueness of order,
    b. Transitivity of order,
    c. Addition for inequalities, and
    d. Multiplication for inequalities.
17. Use absolute value notation.
18. Compute absolute value.

III. Distance

Objective #3: The student will increase in comprehension of length and distance by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

19. Define distance as the unique positive number which corresponds to each pair of distant points.

20. Apply the Ruler Postulate to the computation of distance.

21. Set up a coordinate system for a line.

22. Define point B lying between points A and C.

23. Understand the introductory theorems concerning betweeness of points.

24. Define basic terms in geometry such as:

    a. segment,

    b. endpoints,

    c. length of a segment,

    d. ray,

    e. interior of a ray, and

    f. opposite rays.

25. Represent segments, rays, and lines by appropriate notation.

26. Represent segments, rays and lines by appropriate diagrams.

IV. Lines, Planes, and Separation

Objective #4: The student will increase in comprehension of lines, planes, and space by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

27. Recognize space as the set of all points.

28. Distinguish between collinear and noncollinear points.

29. Distinguish between coplanar and noncoplanar points.

30. Recognize that a plane contains at least three noncollinear points.

31. Recognize that space contains at least four noncoplanar points.

32. Determine minimum conditions for a plane.

33. Draw diagrams as representations of geometric abstractions.

34. Determine the hypothesis and conclusion of a conditional statement.

35. Define a convex set of points.

36. Explain how a line separates a plane into two nonempty convex sets.

37. Explain how a plane separates space into two nonempty, concux sets.

V. Angles and Triangles

Objective #5: The student will apply the properties of angles, triangles, and congruence to solving problems by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

38. Define an angle as the union of two noncollinear rays that have a common endpoint.

39. Identify the vertex and sides of the angle.

40. Recognize an angle as a plane figure.

41. Distinguish between the interior and exterior of an angle.

42. Define a triangle as the union of the three segments which join, in order, three noncollinear points.

43. Identify the vertices, sides, and angles of a triangle.

44. Identify included sides and included angles.

45. Identify opposite angles.

46. Recognize a triangle as a plane figure.

47. Distinguish between the interior and exterior of a triangle.

48. Define the measure of an angle as the number of degrees it contains.

49. Identify a linear pair of angles.

50. Identify adjacent angles as two angles in a plane that have a common side but no interior points in common.

51. Identify supplementary angles as two angles the sum of whose measures is 180 degrees.

52. Define a right anbles as an angle whose measure is 90 degrees.

53. Define perpendicular sets as line, rays, or segments which intersect to form right angles.

54. Identify complementary angles as two angles the sum of whose measures is 90 degrees.

55. Define an acute angle as an angle whose measure is less than 90 degrees.

56. Define an obtuse angle as an angle whose measure is greater than 90 degrees.

57. Recognize congruent angles as angles that have the same measure.

58. Define vertical angles as two angles whose union is of two intersecting lines.

59. Define congruent segments as segments which have the same length.

60. Recognize the reflexive, symmetric, and transitive properties of a relation.

61. Identify an equivalence relation as a relation which possesses the reflexive, symmetric, and transitive properties.

62. Identify corresponding sides and angles of triangles.

63. Define a congruence between triangles as a correspondence in which corresponding sides and corresponding angles are congruent.

64. Recognize the following minimum conditions for determining congruence between triangles:

    a. SAS (Side-Angle-Side)

    b. ASA (Angle-Side-Angle)

    c. SSS (Side-Side-Side)

65. Apply the basic congruence situations in the proofs of original exercises.

66. Prove and apply the Isosceles Triangle theorem.

67. Distinguish among isosceles, equilateral, and scalene triangles.

68. Define a median of a triangle as a segment with one endpoint at a vertex of the triangle and the other endpoint at the midpoint of the opposite side.

69. Define an angle bisector as a segment whose endpoints are the points of intersection of the triangle and a ray that bisects an angle of the triangle.

VI. A Closer Look At Proof

Objective #6: The student will apply the postulates, axioms, and theorems to proofs of mathematical statements by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

70. Examine and discuss the nature of indirect proof.

71. Use indirect proof in original exercises.

72. Distinguish between existence (at least one) and uniqueness (at most one).

73. Examine proofs which involve both existence and uniqueness properties.

74. Prove and apply the theorem which states that given a plane, a line in the plane, and a point on the line, there is exactly one line that is in the given plane, contains the given point, and is perpendicular to the given line.

75. Define the perpendicular bisector of a segment in a plane as the line in the plane that is perpendicular to the segment at its midpoint.

76. Prove and apply the theorem which identifies the perpendicular bisector of a segment in a plane as the set of all points in the plane that are equidistant from the endpoints of the segment.

77. Prove and apply the theorem which states that given a line and point not on the line, there is at most one line that contains the given point and is perpendicular to the given line.

78. Define a right triangle as a triangle that has one right angle.

79. Identify the hypotenuse and legs of a right triangle.

80. Prove and apply the theorem which states that given a line and a point not on it, there is at least one line that contains the given point and is perpendicular to the given line.

81. Introduce auxiliary sets when the need arises.

82. Prove and apply the ASA and SSS congruence theorems.

VII. Geometric Inequalities

Objective #7: The student will apply the properties of geometric inequalities to proofs of mathematical statements by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

83. Define an exterior angle of a triangle as an angle that forms a linear pair with one of the angles of the triangle.

84. Define remote interior angles as those angles of a triangle with which the exterior angle does not form a linear pair.

85. Prove and apply the theorem which states that the measure of an exterior angle is greater than the measure of each remote interior angle.

86. Prove and apply the SAA case of congruence.

87. Prove and apply the H.L. case of congruence in a right triangle.

88. Prove and apply the theorem which states that if two sides of a triangle are not congruent, the angles opposite them are not congruent, and the larger angle is opposite the longer side.

89. Prove and apply the theorem which states that if two angles of a triangle are not congruent, then the sides opposite them are not congruent, and the longer side is opposite the larger angle.

90. Prove and apply the theorem that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

91. Prove and apply the theorem which states that the shortest segment joining a point to a line is the perpendicular segment.

92. Define the distance between a line and a point not on the line as the length of the perpendicular segment from the point to the line.

93. Define an altitude of a triangle as the perpendicular segment from any vertex to the line that contains the opposite side.

94. Prove and apply the theorem which states that if two sides of one triangle are congruent respectively to two sides of a second triangle, and if the measure of the included angle of the first triangle is greater than the measure of the included angle of the second, then the third side of the first triangle is longer than the third side of the second.

95. Prove and apply the theorem which states that if two sides of one triangle are congruent respectively to two sides of a second triangle, and if the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second.

VIII. Perpendicular and Parallel Lines and Planes in Space

Objective #8: The student will apply axioms and theorems to proofs involving paralles and perpendicular lines and planes in space by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

96. Identify a line and a plane as being perpendicular if (1) they intersect and (2) the given line is perpendicular to every line that is in the plane and intersects the given line.

97. Identify a segment or ray as being perpendicular to a plane if (1) it intersects the plane, and (2) the line that contains it is perpendicular to the plane.

98. Prove and apply the theorem which states that if two points, P and Q are each equidistant from two points A and B, then every point between P and Q is equidistant from A and B.

99. Prove and apply the theorem which states that if a line is perpendicular to each of two intersecting lines at their point of intersection, then it is perpendicular to the plane containing them.

100. Prove and apply the theorem which states that for any line and any point of the line there exists a plane perpendicular to the line at the point.

101. Prove and apply the theorem which states that for any plane and any point of the plane there exists a line perpendicular to the plane at the point.

102. Prove and apply the theorem which states that if a line and a plane are perpendicular, then the plane contains every line perpendicular to the given line at its point of intersection with the plane.

103. Prove and apply the theorem which states that for any line and any point of the line there is at most one plane perpendicular to the line at the point.

104. Define the perpendicular bisecting plane of a segment as the plane perpendicular to the segment at its midpoint.

105. Prove and apply the theorem which states that for any plane and any point of the plane there is at most one line perpendicular to the plane at the point.

106. Prove and apply the theorem which states that the perpendicular bisecting plane of a segment is the set of all points equidistant from the endpoints of the segment.

107. Prove and apply the theorem which states that two lines perpendicular to the same plane are coplanar.

108. Prove and apply the theorem which states that for any point and any line there is exactly one plane containing the given point and perpendicular to the given line.

109. Prove and apply the theorem which states that for any point and any plane there is exactly one line containing the given point and perpendicular to the given plane.

110. Define the distance to a plane from a point not on the plane as the length of the perpendicular segment from the point to the plane.

111. Prove and apply the theorem which states that the perpendicular segment to a plane from a point not on the plane is shorter than any other segment from the point to the plane.

112. Define two lines as being parallel if they are coplanar and do not intersect.

113. Define two lines as being skew if they are not coplanar and do not intersect.

114. Prove and apply the theorem stating that two parallel lines lie in exactly one plane.

115. Prove and apply the theorem stating that if two lines in the same plane are both perpendicular to the same line, then they are parallel.

116. Prove and apply the existence theorem about parallelism.

117. Define and identify transversals, alternate-interior angles, corresponding angles, interior angles on the same side of the transversal, and alternate-exterior angles.

118. Prove and apply those theorems which establish that two lines are parallel.

119. Define the Parallel Postulate.

120. Apply the Parallel Postulate to prove other theorems.

121. Prove and apply those theorems stating that if two parallel lines are cut by a transversal, then alternate interior angles are congruent, each pair of corresponding angles are congruent, and any two interior angles are supplementary if their interiors intersect.

122. Prove and apply those theorems stating that in a plane, two lines parallel to the same line are parallel to each other and that if a line is perpendicular to one of two parallel lines it is perpendicular to the other.

123. Relate parallelism to triangles and quadrilaterals.

124. Define a quadrilateral.

125. Classify quadrilaterals and identify the properties of each.

126. Prove and apply the statement that the segment joining the midpoints of two sides of a triangle is parallel to the third side, and the length of the segment is half the length of the third side.

127. Prove and apply those theorems concerned with transversals to several parallel lines.

128. Define parallel planes.

129. Prove and apply the theorems concerned with parallel planes.

130. Prove and apply the theorems concerned with parallels and perpendiculars.

131. Define a dihedral angle.

132. Define and apply all properties of a dihedral angle.

IX. Areas of Polygonal Regions

Objective #9: The student will apply methods of finding areas to a variety of polygonal regions by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

133. Define a function.

134. Define a function from A into B.

135. Define a triangular region.

136. Define a polygonal region.

137. Apply the Area Postulate.

138. Apply the congruence Postulate for Area.

139. Apply the Area Addition Postulate.

140. Apply the Unit Area Postulate.

141. Develop and compute areas of polygonal regions.

142. Prove and apply the Pythagorean Theorem.

143. Prove and apply those theorems concerning the 30-60 right triangle.

X. Similarity

Objective #10: The student will apply properties of similarity to proofs of statements involving similar figures by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

144. Define a sequence, an infinite sequence and a finite sequence.

145. Distinguish among different types of sequences.

146. Define two sequences which are proportional.

147. Define the constant of proportionality.

148. Prove and apply the theorem stating that proportionality of sequences is an equivalence relation.

149. Define similarity or triangles.

150. Apply the algebraic properties of a simple proportion.

151. Prove and apply the basic proportionality theorem.

152. Prove and apply the theorem which states that if a line intersects two sides of a triangle and cuts off corresponding segments that are proportional to the two sides, then the line is parallel to the third side.

153. Prove and apply the AAA similarity theorem.

154. Prove and apply the SAS similarity theorem.

155. Prove and apply the SSS similarity theorem.

156. Prove and apply the theorem which states that for any given right triangle, the three triangles contained in the union of the given triangle and the altitude to the hypotenuse are similar.

157. Prove and apply the theorem which states that if two triangular regions are similar, then the ratio of their areas is the square of the ratio of the lengths of any two corresponding sides.

158. Prove and apply the theorem which states that if 3 parallel lines are cut by two transversals, then the lengths of the segments intercepted on one transversal are proportional to the lengths of the corresponding segments intercepted on the other transversal.

XI. Circles and Spheres

Objective #11: The student will apply the properties of circles and spheres to proofs and problems involving circles and spheres by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

159. Define a sphere with center P and radius R as the set of all points at a distance R from P.

160. Define a circle with center P and radius R as the set of all points in a plane at a distance R from P.

161. Define the diameter of a circle as the number that is twice the radius.

162. Prove and apply the theorem which states that the intersection of a sphere and any plane containing the center of the sphere is a circle with the same center and radius as the sphere.

163. Define the following terms related to circles and spheres:

    a. great circle,

    b. chord,

    c. secant,

    d. diametral chord,

    e. radial segment,

    f. tangent line,

    g. tangent plane, and

    h. point of tangency.

164. Prove and apply the theorem concerning the interior points of a circle.

165. Prove and apply the theorem which states that a line in the plane of a circle is tangent to the circle if and only if it is perpendicular to a radial segment.

166. Prove and apply the theorem which states that the segment joining the center of a circle to a point of the chord bisects the chord if and only if the segment is perpendicular to the chord.

167. Define the distance from the center of a circle (or sphere) to a chord as the distance between the center and the line containing the chord.

168. Prove and apply the theorem which states that for a circle or for two circles of equal radii, two chords are equidistant from the center if and only if the chords are congruent.

169. Prove and apply the theorems concerned with the intersection of planes with a sphere.

170. Define the following terms related to circles:

    a. central angle,

    b. minor arc,

    c. major arc, and

    d. semicircle.

171. Find the degree measures of both major and minor arcs.

172. Prove and apply the arc addition theorem.

173. Define intercepted arcs and inscribed angles.

174. Prove and apply the theorem stating that the measure of an inscribed angle is half the measure of the intercepted arc.

175. Prove and apply the theorem stating that in a circle or in congruent circles, if two chords are congruent, then their corresponding arcs are congruent.

176. Prove and apply the converse of the above theorem.

177. Prove and apply the theorems concerned with the measures of various angles in a circle.

178. Prove and apply the Two Circle theorem.

179. Define the following terms related to circles:

    a. common tangent,

    b. external common tangent,

    c. internal common tangent,

    d. circles tangent internally, and

    e. circles tangent externally.

180. Prove and apply the theorems concerning tangent and secant segments in a circle.

181. Define a polygon and the following related terms:

    a. vertices,

    b. sides,

    c. diagonal,

    d. angles, and

    e. perimeter.

182. Define a regular polygon as one which (a) is pseudo-convex (b) has congruent sides, and (c) has congruent angles.

183. Define an inscribed polygon.

184. Define the area of the region bounded by a regular n-gon is 1/2 aen, where a is the apothem and e is the length of each side.

185. Prove and apply the theorem stating that the ratio, $\frac{C}{d}$, of the circumference to the diameter is the same for all circles.

186. Prove and apply the theorem stating that the area of a circular region of radius R is given by R^2.

187. Define the length of arc A as $\frac{}{180}$ r m (AB).

188. Define the area of sector AB of radius r as $\frac{mAB}{360}$ r^2.

XII. Concurrency and Constructions

Objective #12: The student will apply properties of construction by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

189. Given an angle and a ray, construct an angle, with the given ray as a side, congruent to the given angle.

190. Given an angle, construct the ray that bisects the angle.

191. Given a point on a line in a plane, construct a line perpendicular to the given line at the given point.

192. Given a point outside a line, construct a perpendicular to the line from the point.

193. Given a segment, construct the perpendicular bisector of the segment.

194. Given a point outside a line, construct a line parallel to the given line through the given point.

195. Given an arc of a circle, construct a line that bisects the arc.

196. Given a point on a circle, construct the tangent to the circle at the given point.

197. Given a point outside a circle, construct the tangents to the circle from the point.

198. Given a triangle, circumscribe a circle about the triangle.

199. Given a triangle, inscribe a circle in the triangle.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY THE PRINCIPLES OF MATHEMATICS TO PRACTICAL PROBLEMS BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Numbers

Objective #1: The student will increase in comprehension of numbers by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Express quantity by the use of numbers.
2. Write the integers which are whole numbers.
3. Tell which digits are used to indicate a total number.
4. Determine the difference between face value and place value when working with different numerations.
5. Write the place values of the digits for any number using base 10.
6. Write the place values and numerical value of each digit in a number using base 10.
7. Write the numerical value of a number written in base five.
8. Express a number using words.
9. Round off a number to nearest ten, hundred, thousand, etc.
10. Round off an answer when dealing with money to the nearest dollar.
11. Read a decimal fraction.

12. Express common fractions as decimal fractions.

13. Round off decimal fractions to any particular place value.

14. Express per cent as a decimal fraction.

15. Express decimal fractions as per cent.

16. Express common fractions as per cent.

17. Use a bar graph to determine a title, what is shown on the horizontal axis and vertical axis, and relate the picture to data given.

18. Make a bar graph.

19. Read a broken-line graph.

20. Construct a broken-line graph.

21. Read a picture graph.

22. Read a circle or rectangular graph.

23. Construct a circle or rectangular graph.

II. Statistics

Objective #2: The student will increase in comprehension of statistics by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

24. Find the arithmetic average of a set of numbers.

25. Illustrate the distributive law with respect to the product of two numbers.

26. Set up a frequency table with columns of values, tally marks frequency, and sum.

27. Determine the median of a set of numbers by arranging them in order of size.

28. Determine the mode of a set of numbers.

29. Determine the mean and medium from grouped data.

30. Determine the mid-point of grouped data.

III. Application of Numbers

Objective #3: The student will apply the properties and operations of numbers by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

31. Add numbers horizontally by making use of the commutative law for addition.

32. Add numbers vertically and horizontally making use of the associative law for addition.

33. Estimate the product of two numbers and then compute it.

34. Compute the per cent of a number.

35. Apply the finding of per cent to the weekly income tax of a weekly salary.

36. Illustrate the meaning of a fractional per cent as a fractional part of one per cent.

37. Subtract numbers in a horizontal arrangement.

38. Set up a payroll roster and apply horizontal addition and subtraction to determine net pay.

39. Fill out a currency break-down containing the exact number and kinds of bills and coins needed to meet a particular payroll.

IV. Measurements

Objective #4: The student will increase in knowledge of

measurements by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

40. Measure an object by comparing it with some unit of measure.

41. Use a ruler to measure an object.

42. Illustrate that all measurements are approximate and precision of a measurement depends upon the size of the smallest unit with which the measurement is made.

43. Change a mixed number to an improper fraction.

44. Change an improper fraction to a mixed number.

45. Determine the maximum error in any measure.

46. Change a fraction to an equivalent fraction.

47. Find the prime factors of a number.

48. Find the common factors of two numbers.

49. Simplify a fraction as far as possible.

50. Add two proper fractions.

51. Add a proper fraction and a mixed number.

52. Add a mixed number to a mixed number.

53. Subtract two fractions or mixed numbers.

54. Multiply two fractions and a mixed number.

55. Multiply a proper fraction and a mixed number

56. Multiply two mixed numbers.

57. Divide two proper fractions.

58. Divide a fraction by a mixed number.

59. Divide a mixed number by a mixed number.

60. Determine the accuracy and significant digits in a number.

61. Add two decimal fractions.

62. Subtract two decimal fractions.

63. Compute upper and lower limit of a measure when given the tolerance.

64. Multiply two decimal fractions.

65. Change a common fraction to a decimal fraction.

66. Change decimal fractions to common fractions.

67. Divide a decimal fraction by a whole number.

68. Divide a decimal fraction by a decimal fraction.

69. Illustrate that the metric system is a decimal system.

70. Change metric values to English equivalents.

V. Geometry

Objective #5: The student will apply geometrical concepts to problems of geometry by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

71. Describe and identify a line, plane and point.

72. Define and name a line segment.

73. Define and name an angle and its parts.

74. Define a straight angle, right angle, obtuse angle, acute angle, and a reflex angle.

75. Measure an angle with the use of a protractor.

76. Define perpendicular lines, parallel lines, and the distance between two parallel lines.

77. Define corresponding angles and alternate interior angles.

78. Copy a given angle with a straight edge and compass.

79. Construct a line parallel to a given line through an outside point.

80. Divide a segment into any number of equal segments by using parallel lines.

81. Define congruent triangles.

82. Define corresponding parts of congruent triangles.

83. Define triangles classified according to angles (acute, obtuse, right).

84. Define triangles classified according to sides (scalene, isosciles, and equilateral).

85. Prove triangles congruent by using the S.A.S. theorem.

86. Define vertical angles.

87. Prove triangles congruent by using the A.S.A. theorem.

88. Prove triangles congruent by using the S.S.S. theorem.

89. Define the ratio of two numbers.

90. Apply ratio to the measures of sides of a triangle or triangles.

91. Define a proportion .

92. State the properties of a proporption involving means and extremes of the proportion.

93. Use a proportion for solving problems in which certain kinds of comparisms are made.

94. Define similar triangles.

95. Prove triangles similar by using the AA theorem.

96. Apply the proportions developed from similar triangles to find particular sides of heights of objects.

97. Define inductive reasoning.

98. Apply inductive reasoning to determine the sum of the measures of the angles of a triangle, an exterior angle of a triangle, and the sum of the measures of the angles of a quadrilateral.

99. Define a parallelogram and state its characteristics.

100. Defind deductive reasoning.

101. Test the validity of an argument by using diagrams and deductions.

102. Prove corresponding parts of congruent triangles congruent.

VI. Formulas Dealing with Perimeter, Area and Volume

Objective #6: The student will apply geometrical concepts to problems of perimeter, area, and volume by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

103. Define a formula.

104. Find the perimeter of any polygon.

105. Define the radius of a circle.

106. Find the circumference of a circle.

107. Define π (pi).

108. Find the area of a rectangle.

109. Find the area of a parallelogram.

110. Find the area of a triangle.

111. Find the area of a trapezoid.

112. Find the area of a circle.

113. Find the volume of a rectangular solid.

114. Find the volume of a cylinder.

115. Find the volume of a cone.

116. Find the volume of a sphere.

VII. Algebra

Objective #7: The student will apply abstract reasoning to algebraic properties by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

117. State in words the meaning of particular algebraic expressions.

118. Illustrate the commutative law for addition and multiplication.

119. Translate a word statement into an algebraic expression.

120. Define a variable.

121. Find the value of a numerical expression.

122. Find the relation existing between two variables.

123. Define a statement or equation.

124. Solve an equation in one variable.

125. Solve an equation by division.

126. Solve an equation by suing multiplication.

127. Solve an equation by suing subtraction.

128. Solve an equation by using addition.

129. Solve an wquation requiring more than one operation.

130. Combine like terms in an equation.

131. Solve verbal problems which require the use of equations.

VIII. Signed Numbers

Objective #8: The student will apply the properties of numbers to include signed numbers by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

132. Define a signed number.

133. Represent changes by using signed numbers.

134. Illustrate a number scale.

135. Add signed numbers.

136. Apply the commutative low of addition to signed numbers.

137. Find the absolute value of a number, signed or unsigned.

138. Subtract signed numbers.

139. Multiply signed numbers.

140. Combine like terms involving signed numbers.

141. Apply the distributive law to expressions with parentheses.

142. Solve equations involving signed numbers.

143. Graph an equation which is a function.

144. Read a graph which represents an equation.

145. Graph equations which involve signed numbers.

146. Read order pairs from a graph of an equation involving signed numbers.

IX. Application of Formulas

Objective #9: The student will apply formulas to problem solving situations by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

147. Apply known formulas and given formulas to determine the solution of a physical problem.

148. Name the sides of a right triangle.

149. State the relation between the acute angles of a right triangle.

150. State the Pythagorean Theorem.

151. Find the missing side of a right triangle when given the measure of the other two sides.

152. Find the perfect square of a number.

153. Find the square root of a number from a table of squares.

154. Find the square root by using division.

155. Determine the actual length of an object by using a given scale.

156. Change the actual measurements of an object to those of a smaller represnetation.

157. Find the perimeter and area of an object of a scale drawing.

158. Find the actual distance between two points on a map.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY THE PRINCIPLES OF NUMBERS AND POLYNOMIALS BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Numbers and Sets

Objective #1: The student will apply the principles of numbers and sets by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Determine the elements of a set.
2. Represent a set using the roster method.
3. Represent a set using the rule method.
4. Find subsets of a set.
5. Recognize the set of no elements as the null set.
6. Recognize a finite set.
7. Recognize an infinite set.
8. Recognize equal sets.
9. Recognize equivalent sets.
10. Set up a one to one correspondence between the elements of equivalent sets.
11. Identify the set of counting numbers.
12. Determine thesuccessor of any whole number.
13. Determine multiples of a number.
14. Identify odd or even numbers.

15. Use the number line for indentifying the relations "is equal to," "is not equal to," "is greater than," and "is less than."

16. Identify the rational numbers of arithmetic.

17. Distinguish between a numeral and a number.

18. Simplify numerical phrases.

19. Follow the order of operations for ungrouped or grouped numerical phrases.

20. Use grouping symbols in numerical phrases.

21. Use symbols and numerals to form sentences.

22. Determine that a sentence is true or false.

23. Recognize a variable as a symbol that may be replaced by any element of a specified set.

24. Recognize the domain of the variable.

25. Determine the value of an open phrase.

26. Represent word phrases by open phrases.

27. Use variables to represent number patterns.

28. Determine the truth number for an open sentence.

29. Determine the truth set of an open sentence.

30. Use exponents to abbreviate phrases involving a repeated base.

31. Graph truth sets on the number line.

32. Determine truth values for compound sentences using "and" and "or" as connectives.

33. Determine the intersection of sets.

34. Graph compound sentences whose connectives are "and" or "or"

35. Use the addition and multiplication properties of zero.

36. Use the multiplication property of one.

37. Recognize that the identity elements for addition and multiplication are zero and one.

38. Recognize binary operations.

39. Recognize a closure property.

40. Recognize and use the commutative property for addition.

41. Recognize and use the associative property for addition.

42. Recognize and use the commutative property for multiplication.

43. Recognize and use the associative property for multiplication.

44. Recognize and use the distributive law for multiplication over addition.

II. Operations and Polynomials

Objective #2: The student will apply properties and operations of the real number system to algebraic expressions by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

45. Simplify phrases by using properties of numbers.

46. Find the product of a monomial and a monomial.

47. Find the product of a monomial and a binomial or polynomial.

48. Find the product of two binomials.

49. Find factors of a number.

50. Test divisibility by 2, 3, 4, 5.

51. Identify a prime number.

52. Identify a composite number.

53. Factor a number into a product of primes.

54. Recognize that prime factor is unique.

55. Apply exponents as an abbreviation for repeated factors.

56. Use fractions to name numbers.

57. Multiply fractions.

58. Find the least common multiple of two or more counting numbers.

59. Add fractions.

60. Find the reciprocal of a number.

61. State that a ratio is the comparison of two numbers by division.

62. Write a proportion.

63. Determine a true proportion.

64. Determine equivalent proportions.

65. Apply proportions in solving problems.

66. Convert fractions to per cent.

67. Apply per cent in solving problems.

68. Identify units of measure.

69. Determine greatest possible error.

70. Determine precision of a measurement.

71. Convert among standard units of measure.

72. Find area of certain plane figures.

73. Define square units of measure.

74. Find the volume of certain solid figures.

75. Define cubic units of measure.

76. Find perimeters of certain plane figures.

77. Convert word phrases to open phrases.

78. Form open sentences from word sentences.

79. Solve number problems.

80. Solve problems involving perimeters and areas.

81. Solve problems in costs.

82. Solve distance problems.

83. Classify subsets of numbers of the real numbers.

84. Perform operations of addition, subtraction, multiplication, and division with the set of real numbers.

85. Recognize order in the set of real numbers.

86. Apply the concepts of absolute value with respect to distances on number lines.

87. Apply properties of addition and multiplication developed on counting numbers to the set of real numbers.

88. Determine equivalent open sentences.

89. Use the addition property of equality to solve equations.

90. Use the multiplication property of equality to solve equations.

91. Add polynomials.

92. Multiply polynomials.

93. Subtract polynomials.

94. Simplify complex fractions.

95. Determine the probability of a favorable outcome from given conditions of uncertainty.

96. Recognize the difference between experimental probability and mathematical probability.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY ABSTRACT REASONING TO THE REAL NUMBER SYSTEM AND ALGEBRAIC EXPRESSIONS BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Real Numbers and Order

Objective #1: The student will increase in knowledge of order and the principles of real numbers by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Use the symbol for the opposite of a number.
2. Write the absolute value of a number.
3. Add, subtract, multiply, and divide:
    a. two positive numbers,
    b. two negative numbers, and
    c. one positive and one negative number
4. Recognize and identify the following properties of the real numbers:
    a. closure,
    b. identity,
    c. inverse,
    d. commutative,
    e. associative, and
    f. distributive.
5. Recognize and combine like terms.

6. Use addition and multiplication properties to solve simple open sentences.

7. Solve simple verbal problems.

8. Solve and graph simple inequalities.

9. Recognize and identify the following properties of order:

   a. comparison property,

   b. order property of opposites,

   c. transitive,

   d. addition, and

   e. multiplication.

10. Solve and graph inequalities using the above properties.

11. Solve and graph compound inequalities.

II. Powers and Roots

Objective #2: The student will increase in knowledge of powers and roots by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

12. Identify and differentiate between exponent, coefficient, base and power.

13. Apply order of operations to simplify expressions.

14. Apply the following properties of exponents:

   a. $x^a \, x^b = x^{a+b}$,

   b. $\frac{x^a}{x^b} = x^{a-b}$ where $x \neq 0$,

   c. $x^{-a} = \frac{1}{x^b}$ where $x \neq 0$,

d. $x^0 = 1$ where $x \neq 0$,

e. $(xy)^n = x^n y^n$,

f. $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ where $y \neq 0$,

g. $(x^a)^b = x^{ab}$.

15. write a real number in scientific notation.

16. Find the square root of a number.

17. Find the square of a number.

18. Recognize perfect squares.

19. Determine the prime factorization of a number.

20. Differentiate between rational and irrational numbers.

21. Multiply numbers in radical form.

22. Simplify radical numbers and expressions.

23. Combine radical numbers and expressions.

24. Divide numbers in radical form.

25. Rationalize the denominator.

26. Approximate square root by division.

III. Polynomials

Objective #3: The student will increase in knowledge of operations and properties of polynomials by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

27. Recognize and differentiate between monomial, binomial, trinomial, and polynomial.

28. Identify the coefficient of a monomial.

29. Identify the degree of a polynomial.

30. Identify the factor common monomial factors.

31. Factor quadratic trinomials.

32. Factor quadratic polynomials.

33. Identify and factor the binomial called difference of two squares.

34. Identify and factor perfect trinomial squares.

35. Factor polynomials completely.

IV. Relations and Functions

Objective #4: The student will increase in knowledge of relations, functions, and their graphs by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

36. Recognize the following terms:

    a. coordinate of a point,

    b. ordered pair,

    c. x-axis,

    d. y-axis,

    e. origin,

    f. coordinate plane, and

    g. quadrant.

37. Graph an ordered pair on the coordinate plane.

38. Define and identify relations.

39. Graph relations.

40. Define and identify functions.

41. Define and state the domain and range of a set.

42. Graph functions.

43. Identify linear functions.

44. Use a table of values to graph a linear function.

45. Find the y-intercept.

46. Find the slope of an equation.

47. Recognize the slope-intercept form of an equation.

48. Identify parallel lines.

49. Find the slope of an equation by using the slope formula.

50. Recognize the x-intercept.

51. Apply linear functions to practical situations.

52. Write linear functions from a table of values using the rule method.

V. Equations and Inequalities

Objective #5: The student will increase in knowledge of equations and inequalities by graphing and solving sentences by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

53. Recognize an equation in standard form.

54. Write an equation in the satandard form ax+by+c=o.

55. Identify systems of linear equations.

56. Solve systems of linear equations by graphing.

57. Identify equivalent systems.

58. Solve systems by substitution.

59. Solve systems by linear-combination.

60. Solve special systems of equations such as parallel lines and coinciding lines.

61. Use systems of equations to solve verbal problems.

62. Write linear inequalities in slope-intercept form.

63. Graph simple linear inequalities involving, $>$, $<$, $\geq$, and $\leq$.

64. Graph compound inequalities with union and intersections.

65. Graph systems of linear inequalities.

66. Apply inequalities to practical situations.

VI. Rational Number Operations

Objective #6: The student will increase in knowledge of the operations of rational numbers by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

67. Identify rational expressions.

68. Simplify rational expression using $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ where $b \neq 0$ and $d \neq 0$.

69. Multiply rational expression with:

    a. monomial numerators and denominators,

    b. binomial numerators and denominators, and

c. trinomial numerators and denominators.

70. Apply the quotient rule to rational expression $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.

71. Divide rational expressions.

72. Find the common name for rational expressions using

a. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ b. $\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}$ where $c \neq 0$.

73. Find the least common multiple of two or more polynomials.

74. Find the sums and differences of rational expressions with:

a. monomial denominators,

b. binomial denominators, and

c. trinomial denominators.

75. Solve equations involving rational expressions.

76. Simplify a complex fraction.

77. Apply rational expressions to practical situations.

VII. Quadratic Functions

Objective #7: The student will increase his ability to apply quadratic functions by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

78. Identify quadratic functions.

79. Find truth sets of the quadratic functions.

80. Graph quadratic functions using a table of values.

81. Recognize the relationship between:

    a. the graph of $y=ax^2$ and the value of a,

    b. the graph of $y=a(x-h)^2$ and the value of h,

    c. the graph of $y=a(x-h)^2+h$ and the value of h.

82. Determine the vertex and axis of symmetry of the parabola $y=a(x-h)^2+h$.

83. Recognize the standard form of a quadratic polynomial.

84. Write quadratic polynomials in standard form.

85. Find the greatest and least value of a polynomial function.

86. Find the y-intercept of a quadratic polynomial.

87. Sketch the graph of a polynomial quadratic using the y-intercept, vertex, and axis of symmetry.

88. Find the zeros of quadratic polynomials from a graph.

89. Recognize standard form of quadratic equations.

90. Write quadratic equations in standard form.

91. Find the truth set of a quadratic function by

    a. factoring, and

    b. quadratic formula.

92. Recognize the relationship between the Discrimenant, b -4ac, and the nature of the roots and the number of roots.

93. Apply quadratic equations to practical situations.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY HIS KNOWLEDGE OF MATHEMATICS TO COMPUTER PROGRAMMING BY COMPLETING THE FOLLOWING SUGGESTED ACTIVITIES TO THE SATISFACTION OF THE TEACHER.

I. Fortran

Objective #1: The student will increase his comprehension of the language and structure of Fortran by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Define Fortran.
2. Define a computer program.
3. Define a source program.
4. Define an object program.
5. Define data.
6. Define coding.
7. Define a coding sheet.
8. Define a statement number.
9. Describe a punched card.
10. Define compiler program.
11. Define control cards.
12. Define algorithm.
13. Write a comment card.
14. State the Fortran character set.
15. Write basic formulas in Fortran characters.
16. Apply dilimiters to Fortran statements which involve addition, subtraction, division, multiplication, and exponentials.

17. Write words, numbers, names, and expressions from the Fortran character set.

II. Statements

Objective #2: The student will apply his knowledge of statements to computer programs by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

18. Write an arithmetic statement which does not require data.

19. Write an arithmetic statement which requires data.

20. Write a series of statements to solve basic formulas of math and science.

21. Write a Read Statement which involves integer variables to be used as a data set reference number.

22. Write a Read Statement which involves an integer variable and a statement number which is Format statement.

23. Write a Read Statement which represents a list of one or more variable names and one or more array names, separated by commas.

24. Write a Format statement which will direct the computer to read data from punched cards.

25. Write numbers involving fixed point variables.

26. Write a Format which allows the reading of more than one card.

27. Write Read Statements using the Integer Format code.

28. Write Format Statements which involve Floating point numbers.

29. Write statements, both Read and Format, which involve the E and D Exponent forms.

30. Write a WRITE Statement that involves integer variables which are used as a data set reference number.

31. Write a WRITE Statement that involves an integer variable and a statement number which is a Format Statement.

32. Write a WRITE Statement which represents a list of one or more variable names and one or more array names, separated by commas.

33. Write a Format Statement for the write instruction which will direct the computer to write variables which are integers or floating point.

34. Write a Format Statement for the write instruction which will space and label the output on the printer of any given program.

35. Write a Format Statement which will cause the printer to start printing on a new page.

III. Arrays and Subscripted Variables

Objective #3: The student will apply his knowledge of arrays and subscripted variables to computer statements by completing the following suggested activities to the satisfaction of the teacher.

Activities:

36. Write a DIMENSION Statement which assignes names to one or more arrays.

37. Write a subscripted variable with one, two, and three dimension arrays.

38. Write a Format statement which causes the computer to read or write the subscripted variable.

39. Write an indexed subscripted variable which allows the use of only part of an array.

40. Write programs which will involve subscripted variables.

IV. Branching and Looping

Objective #4: The student will apply his knowledge of computer programming by creating statements involving branching and looping by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

41. Write a Flow Chart which will illustrate the operations involved in any given program.

42. Write an IF statement which causes the computer to branch to one of three statement numbers.

43. Write a GO TO Statement which causes the computer to branch to more than three statements.

44. Write an Unconditional GO TO Statement which allows the computer to return from a branch and continue the program at a desired statement.

45. Write the DO Statement which causes the computer to repeat in a loop, operations of desired statements.

46. Write programs which involve the DO Statement to span over an entire program.

47. Write a program which makes use of the continue Statement.

48. Write a program which involves more than one DO Statement.

V. Subroutine Subprograms

Objective #5: The student will apply his knowledge of computer programming by completing the following suggested activities on subroutine subprograms to the satisfaction of the teacher.

Activities:

The student will be able to:

49. Write the subroutine statement which allows the computer to solve a portion of a problem.

50. Write the Call Statement which directs the computer to branch to the subprogram.

51. Write programs which involve subroutines.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY HIS KNOWLEDGE OF THE PRINCIPLES OF MATHEMATICS TO THE OPERATION OF THE SLIDE RULE BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. The Decimal Point

Objective #1: The student will apply his knowledge of the principles of decimal notation by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Locate the decimal point by inspection.
2. Eliminate the decimal point in products and quotients.
3. Write a number in scientific notation.
4. Apply the laws of exponents to multiplication and division.
5. Multiply numbers using scientific notation.

II. The Scales

Objective #2: The student will apply his knowledge of the scales on a slide rule to performing the operations of mathematics by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

6. Name the physical parts of the slide rule.
7. Determine the difference between a uniform and non-uniform scale.
8. Describe the primary and secondary division marks on the C and D scales.
9. Read the C and D scales for different hairline settings.

10. Determine the accuracy of the different length slide rules.

11. Read the L scale.

12. Use the L scale to check the C and D readings.

13. Realize that a slide rule adds and subtracts lengths.

14. Multiply two numbers using the C and D scales.

15. Use either index when multiplying two numbers.

16. Divide two numbers using the C and D scales.

17. Locate the decimal point whether multiplication or division is performed.

18. Alternate the multiplication and division operations, regardless of which is chosen as the initial operation.

19. Realize that it may be impossible at this time to alternate the division and multiplication operations on the C and D scale.

20. Identify the CI scale as being identical to the C scale except that it reads from right to left.

21. Find the reciprocal of a number using the C and CI scales.

22. Find the reciprocal of a number using the D and DI scales.

23. Use the CI scale for division.

24. Use the CI scale for multiplication.

25. Identify the important feature of the CI scale as its treatment of multiplication as division and vice versa.

26. Use the CI scale to continue division.

27. Use the CI scale to continue multiplication.

28. Use the CI scale to continue a series of operations when a setting may go off the rule.

29. Describe the CF, D, F, CIF scales and tell how they are related to C, D, and CI scales.

30. Multiply on the folded scales.

31. Shift from the regular to the folded scales during a combined or continued operation.

32. Shift from the folded to the regular scales during a combined or continued operation.

33. Multiply and divide by π

III. Ratio and Proportion

Objective #3: The student will apply his knowledge of the concepts of the slide rule to solving problems involving ratio and proportion by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

34. Define a ratio.

35. Define a proportion.

36. Solve a proportion using the C-D (CF-DF) scales.

37. Solve an extended proportion using the C-D (CF-DF) scales.

IV. Powers and Roots

Objective #4: The student will apply his knowledge of the slide rule in problems of powers and roots by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

38. Describe the A and B scales and relate their difference with the C and D scales.

39. Square a number.

40. Find the square root of a number between 1 and 100.

41. Find the square root of a whole number greater than 100 or any number greater than 1.

42. Find the square root of any number less than 1.

43. Find the square root of a number using the powers of ten method.

44. Describe the K scale with its three sections.

45. Cube a number using the K scale.

46. Find the cube root of a number between 1 and 1000.

47. State the general procedure for finding cube roots.

48. Find the cube root of any number greater than 1 by using the power of ten method.

49. Find the cube root of any number less than 1 by using the power of ten method.

50. Find the 3/2 root of any number.

51. Find the 2/3 root of any number.

52. Divide a number by the square root of another number using the C, D, A, and B scales.

53. Multiply a number by the square root of another number using the C, D, A, and B scales.

54. Multiply any number by the square of another number.

55. Divide any number by the square of another number.

56. Find the area of a circle using the A and B scales.

57. Multiply a number by the square root of another number using the A and B scales.

58. Divide a number by the square root of another number using the A and B scales.

59. Multiply or divide expression with square roots.

60. Multiply or divide expressions with square roots and squares.

61. Find the product or quotient of numbers involving squares, square roots or combinations of any number.

V. The Trigonometric Functions

Objective #5: The student will apply his knowledge of the slide rule in problems involving trigonometric functions by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

62. State and identify the six basic trig functions and their reciprocal relations. (sin X, cos x, tan x, csc x, cot x, and sec x)

    sin X = 1/csc x

    cos x = 1/sec x

    tan x = 1/cot x

63. Identify the complementary relations.

    (cos x = sin (90-X))

    (cot x = tan (90 -x))

    (csc x = sec (90-X))

64. Find the sin of X, where X is in degree measure.

65. Place the decimal point when using the S and ST scales.

66. Find the cos of X, when X is in degree measure.

67. Place the decimal point when finding the cosine on the S and ST scales.

68. Find the tan of X when X is a measure less than 45°.

69. Find the tan of X when X is greater than 45°.

70. Place the decimal point for the tan of X for any value of X.

71. Find the cotangent of X.

72. Find the secant of X.

73. Find the cosecant of X.

74. Multiply a number and any of the six basic trig functions.

75. Divide a number by any of the six trig functions.

76. Describe the relation between degree measure and radian measure.

77. Convert from degree measure to radian measure and vice versa using the ST scale.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY HIS KNOWLEDGE OF THE PRINCIPLES OF MATHEMATICS IN AN INTRODUCTORY COLLEGE MATHEMATICS COURSE BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Sets, Relations, and Functions

Objective #1: The student will increase his comprehension of sets, relations, and functions by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Define a set as any well-defined collistion of objects.
2. Read the notation $a \in A$ as "a belongs to A" or "a is an element of A."
3. Distinguish between finite and infinite sets.
4. Provide examples of finite and infinite sets.
5. Describe a set by the tabulation method.
6. Describe a set by the defining-property method.
7. Recognize a mapping of a set A "into" a set B as a matching procedure that assigns to each element "$a \in A$" a unique element "$b \in B$."
8. Read the notation $a \rightarrow a'$ as "a maps into a'."
9. Define the set A as the domain of the mapping, the set B as the codomain of the mapping, and the set of images as the range of the mapping.
10. Recognize a mapping of a set A "onto" a set B as one in which the range is equal to the codomain.
11. Compute the cardinal number of a set.
12. Recognize two sets A and B as being equal when each element of A is an element of B and each element of B is an element of A.

13. Recognize B as a subset of A if and only if each element of B is an element of A.

14. List the subsets of a given set.

15. Define a set B as a proper subset of set A if and only if B is a subset of A and at least one element of A is not an element of B.

16. Define the universe (U) as the complete set or largest set for a particular discussion.

17. Find the solution set for a given set selector.

18. Define the intersection of A and B, written A ∩ B, as the set of elements common to both A and B.

19. Define the union of A and B, written A ∪ B, as the set of elements in A or B, or in both.

20. Construct Venn Diagrams for various situations.

21. Define the null set or empty set as the set which contains no elements.

22. Recognize disjoint (mutually exclusive) sets as two sets having an empty intersection.

23. Define and identify denumerable sets.

24. Identify a conjunction as a compound sentence using the connective "and."

25. Identify a disjunction as a compound sentence using the connective "or."

26. Identify a negation as a sentence involving "not."

27. Construct truth tables for:

    a. conjunction,

    b. disjunction, and

    c. negation.

28. Construct truth tables for sentences involving combinations of conjunctions, disjunctions, and negations.

29. Identify an implication as a sentence of the form "if p then q."

30. Construct the converse, inverse, and contropositive of a given implication.

31. Use the universal quantifier in writing open sentences.

32. Use the existential quantifier in writing open sentences.

II. Real Numbers and Conditions

Objective #2: The student will apply his knowledge of the properties of real numbers by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

33. Investigate number systems as to the following properties:

    a. Closure,

    b. Commutativity,

    c. Associativity, and

    d. Distributive Principle

34. Establish relationships between the various number systems.

35. Establish a one-to-one correspondence between the real numbers and the points of a coordinate line.

36. Summarize the properties of the Real Number Septem.

37. Prove certain theorems concerning real numbers.

38. Perform the prime factorization of polynomials.

39. Explore the concept of order as related to a number line.

40. Distinguish between the inclusive and exclusive "or."

41. Solve algebraic inequalities.

42. Apply the concept of absolute value.

43. Graph sentences involving absolute value.

44. Investigate the concept of Mathematical Induction.

45. Apply Mathematical Induction to proofs.

46. Identify a field and provide examples.

47. Distinguish between upper bounds and lower bounds.

48. Explore the Completeness Property as related to the set of real numbers.

49. Identify denumerable sets.

III. Ordered Pairs, Cartesian Product and Relations

Objective #3: The student will increase his comprehension of a Cartesian co-ordinate plane by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

50. Write ordered pairs.

51. Graph ordered pairs on a coordinate plane.

52. Form a Cartesian Product.

53. Define a relation as a subset of a Cartesian Product.

54. Identify the domain of a relation as the set of all first components of the ordered pairs in the ordered pairs in the relation.

55. Identify the range of a relation as the set of all second components of the ordered pairs in the relation.

56. Identify an equivalence relation as a relation which exhibits the following properties:

    a. Reflexive,

    b. Symmetric, and

    c. Transitive.

57. Provide examples of relations which may be classified as equivalence relations.

58. Identify pairs of complementary relations.

59. Construct the inverse of a relation.

IV. Relations and Functions

Objective #4: The student will apply his knowledge of the principles of relations and functions to graphing by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

60. Graph examples of relations on a coordinate plane.

61. Graph inequalities in two variables.

62. Graph relations involving compound conditions.

63. Define a function as a relation in which no two ordered pairs have the same first component.

64. Use functional notation to evaluate a function at a particular value of X.

65. Identify a neighborhood or a deleted neighborhood of X.

66. Apply the operations of union and intersection to relations and functions.

67. Perform the operations of addition, subtraction, multiplication, and division with functions.

68. Construct the inverse of a function.

69. Perform the composition of two functions.

V. Numerals and Numeration

Objective #5: The student will apply his knowledge of the properties and operations of the real number system by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

70. Distinguish between numeral and number.

71. Represent numbers in the Roman system of numeration.

72. Represent numbers in the Egyptian system of numeration.

73. Represent numbers in the decimal system of numeration.

74. Write expanded numerals in the decimal system of numeration.

75. Represent numbers in non-decimal systems of numeration systems.

76. Perform operations of addition, subtraction, multiplication, and division in non-decimal numeration systems.

77. Represent fractions in non-decimal numeration systems.

78. Define a natural number.

79. Perform operations of addition and multiplication involving natural numbers.

80. Perform operations of subtraction and division of natural numbers when a result exists.

81. Determine the property of closure when it exists for an operation on the set of naturals.

82. Demonstrate the commutative laws for addition and multiplication for naturals.

83. Demonstrate the distributive law for multiplication over addition for naturals.

84. Demonstrate the associative laws for addition and multiplication for naturals.

85. Demonstrate that the addition algorithm depends on the commutative andassociative laws for addition.

86. Demonstrate that the multiplication algorithm depends on the distributive law of multiplication over addition.

87. Identify prime numbers.

88. Factor any composite number into a product of primes.

89. Find the least common multiple for two or more natural numbers.

90. Find the greatest common factor for two or more natural numbers.

91. Identify even natural numbers.

92. Identify odd natural numbers.

93. Find figurate numbers including:

    a. oblong,

    b. rectangular,

    c. triangular,

    d. square, and

    e. cube numbers.

94. Determine divisibility by the numbers 3, 4, 5, 6, 8, 9, and 10.

95. Check the operation of addition by casting out nines.

VI. Mathematical Systems

Objective #6: The student will apply his knowledge of the principles of the real number system to any mathematical system by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

96. Examine and construct mathematical systems other than natural numbers, including

    a. clock authmetic,

b. modular arithmetics, and

c. mathematical systems without numbers.

97. Determine operational identity for a set.

98. Determine operational inverses for a set.

99. Determine commutative porperty for a set and operation.

100. Determine an associative property for a set and operation.

101. Determine that a set and an operation defined on a set form a group or an abelian group.

102. Determine a subgroup.

103. Determine a distributive law for a set and two operations defined on the set.

104. Determine that a set and two operations defined on the set form a field.

105. Determine that a set and two operations defined on the set form a ring.

VII. Mathematical Reasoning and Creativity

Objective #7: The student will be able to analyze mathematical data for proofs by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

105. Arrive at a conclusion inductively through examining certain samples.

107. Arrive at a conclusion through deductive reasoning.

108. Perform proofs in a mathematical system.

109. Use diagrams to illustrate correct reasoning.

110. Establish the validity of a statement.

111. Distinguish between a statement and a sentence.

112. Use variables, constants, and symbols in sentences.

113. Identify a replacement set.

114. Identify a solution set as a subset of some replacement set.

115. Express sets using set notation.

116. Identify set notation.

117. Represent set relations symbolically, including:

    a. intersection,

    b. subset (proper or improper),

    c. union, and

    d. complement.

118. Represent the following set relations using diagrams:

    a. subset,

    b. intersection,

    c. union, and

    d. complement.

119. Determine universal sets.

120. Determine equality for sets.

121. Determine the cardinal numbers of sets.

122. Determine a one-to-one correspondence.

123. Define a number in terms of sets.

124. Determine truth value for compound statements, spicifically

    a. the conjunction,

    b. the disjunction,

    c. the conditional, and

    d. the biconditional.

125. Form the converse, inverse and contrapositive of a given implication.

126. Form the negation of a given statement.

127. Determine truth value for compound sentences using more than one connective.

128. Prove a compound sentence is a tautology by a truth table.

129. Form negations of compound sentences.

130. Write a definition.

131. Structure a statement using a quantifier (universal or existential) and a sentence.

132. Structure a statement using more than one quantifier.

133. Form the negation of quantified statements.

134. Use sentences to describe sets.

135. List the characteristics of Boolean Algebras.

136. Use rules of inference to arrive at desired conclusions including:

    a. substitution,

    b. niodus ponens,

    c. tautologies, and

    d. rule of unditional proof.

137. Test the validity of an argument.

138. Prove certain theorems about groups.

139. Prove certain theorems about fields.

140. Write an informal proof.

141. Use mathematical induction to form a conclusion.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY HIS KNOWLEDGE OF ALGEBRAIC PRINCIPLES BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Algebraic Expressions

Objective #1: The student will apply his knowledge of algebraic expressions by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Identify a given numeral as natural, whole, rational, or irrational.
2. Simplify a given numerical phrase using properties of numbers.
3. Describe the structure of the real number system.
4. Express a given periodic decimal as a rational number.
5. Find the solution set of a linear equation.
6. Recognize equivalent equations.
7. Arrive at an equivalent equation from a given linear equation to find the solution set by transformation.
8. Find the distance between two points from the coordinates of the points.
9. Derive the distance formula.
10. Compute expressions involving positive integral exponents.
11. Compute expressions involving negative integral exponents.
12. Compute expression involving rational exponents.
13. Relate fractional exponents and radicals.

14. Express a radical in simplest form.

15. Perform the operations of addition, multiplication, division, and subtraction with radicals.

16. Simplify expressions involving rational exponents.

17. Factor algebraic expressions such as:

    a. common monomial factoring,

    b. common binomial factoring,

    c. difference between 2 perfect squares,

    d. perfect trinomial square,

    e. general trinomial, and

    f. an + bn.

18. Perform operations involving fractions.

19. Simplify algebraic fractions.

20. Add and subtract algebraic fractions.

21. Simplify complex fractions.

22. Solve fractional equations in one variable.

II. Quadratic Equations

Objective #2: The student will apply his knowledge of algebraic principles to quadratic equations by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

23. Solve a quadratic equation using the factoring method.

24. Solve a quadratic equation by completing the square.

25. Solve a quadratic equation using the quadratic formula.

26. Graph a quadratic equation.

27. Determine the nature of the roots of a quadratic equation by observing the discriminant.

28. Relate the sum and product of the roots to a quadratic equation in standard form.

29. Find the quadratic equation given the roots of a quadratic equation.

III. Logarithms

Objective #3: The student will apply his knowledge of logarithms to the appropriate mathematical operations by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

30. Express a logarithmic expression as an exponential expression.

31. Apply exponential laws to properties of logarithms.

32. Apply properties of logarithms to any base.

33. Solve equations involving logarithms.

34. Explain the development of the common logarithms.

35. Use a table of mantissas to find the common logarithm of a real number.

36. Use a table of mantissas to find antilogarithm.

37. Use the nettas of linear interpolation to approximate mantissas and antilogarithms not found in a table.

38. Use common logarithms to perform the operations of multiplication, division, raising to powers and extracting roots.

39. Solve exponential equations using common logarithms.

IV. Mathematical Sequences

Objective #4: The student will apply his knowledge of mathematical sequences to problem solving by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

40. Identify an arithmetic sequence of numbers.

41. Find the nth term of an arithmetic series.

42. Find the sum of the first n terms in an arithmetic series.

43. Insert arithmetic means between two extremes.

44. Identify a geometric sequence of numbers.

45. Find the nth term of a geometric series.

46. Find the sum of the first n terms of a geometric series.

47. Insert geometric means between two extremes.

V. Statistics and Probability

Objective #5: The student will apply his knowledge of statistics and probability to problem solving situations by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

48. Identify permutations.

49. Solve problems involving permutations.

50. Identify combinations.

51. Solve problems involving combinations.

52. Find the probability an event will occur.

53. Find the probability an event will not occur.

54. Find the probability two or more events will occur.

55. Identify mutually exclusive events.

VI. Theory of Equations

Objective #6: The student will apply the theory of equations to solving problems by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

56. Identify systems of equations in two or three variables.

57. Classify systems as linear-linear, linear-quadratic or quadratic-quadratic systems.

58. Find solution sets for systems of equations using the multiplication-addition method or the substitution method

59. Solve systems of equations graphically.

60. Solve systems of inequalities algebraically.

61. Solve systems of inequalities graphically.

62. Solve systems of equations using determinants.

63. Use synthetic division to divide a polynomial by a binomial.

64. Use synthetic division to factor a polynomial.

65. Use the Factor Theorem to factor a polynomial.

66. Use the remainder Theorem to divide a polynomial by a binomial.

67. Graph equations of a higher than second degree.

VII. Curve Sketching

Objective #7: The student will apply his knowledge of geometric interpretation to an algebraic

situation by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

68. Graph a linear equation by the slope-intercept method.

69. Graph a linear equation by using the x and y-intercept method.

70. Find the y-intercept and x-intercept of a graph.

71. Graph a parabola.

72. Determine the axis of symmetry of a parabola.

73. Determine the translation of axis of a parabola.

74. Find the maximum or minimum points of a parabola.

75. Solve verbal problems involving maximum and minimum points of a parabola.

76. Graph a circle.

77. Determine the Translation of Axis for a circle.

78. Graph an ellipse.

79. Determine the translation of axis of an ellipse.

80. Graph a Hyperbola.

81. Determine the translation of axis of a hyperbola.

82. Graph an exponential function.

83. Graph a linear inequality.

84. Graph the intersection and union of linear inequalities.

85. Graph inequalities of the four conics.

86. Determine the discontinuity of a function.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY HIS KNOWLEDGE OF THE PRINCIPLES OF GEOMETRY BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Triangles and Quadrilaterals

Objective #1: The student will apply his knowledge of the properties of triangles and quadrilaterals to theorems and proofs by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Define basic terms which are used in the study of geometry, such as: midpoint, angle bisector, perpendicular, right angle, acute, adjacent angles, vertical angles, complementary angles, and supplementary angles.
2. Classify triangles by aides.
3. Classify triangles by angles.
4. Define and apply lines and segments associated with triangles, such as altitude, median, and perpendicular bisector.
5. Solve numerical problems involving angles.
6. Define exiom and postulate and know the important axioms and postulate related to segments and angles.
7. Know the definition of Theorem and know those theorems which apply to vertical supplementary, and complementary angles.
8. Select the hypothesis (given) and conclusion (to prove) from a geometric statement.
9. Write the converse of a theorem.
10. Apply those theorems concerned with isosceles triangles.
11. Apply those theorems concerned with parallel lines.

12. Apply those theirems concerned with proving triangles congruent.

13. Learn the definitions and properties of the various types of quadrilaterals.

14. Apply the concept of distance.

15. Know and prove when a quadrilateral in a parallelagram, rectangle, square, or rhombus.

16. Identify the properties of all the quadrilaterals.

17. Know those theorems concerning distance with perpendicular lines and bisectors.

18. Know those theorems which prove lines are parallel.

19. Know and apply those theorems and corollaries which concern the measures of the angles of a triangle or quadrilateral.

20. Apply those theorems and definitions which concern the exterior angles of a triangle.

21. Find the measure of an interior or exterior angle of a regular polygram.

II. Circles

Objective #2: The student will apply his knowledge of theorems concerning circles by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

22. Identify and name the various parts of a circle and segments or lines associated with one or more circles.

23. Understand the relationships between central angles and arcs.

24. Explain the difference between the length of an arc and the measure of an arc.

25. Distinguish between inscribed and circumscribed polygrams.

26. Apply the theorems and corollaries concerning inscribed angles of a circle.

27. Use the term secant ray and tangent ray in relations to describing the positions of angles to circles.

28. Apply those theorems concerning chords, secants, and tangents.

III. Ratio and Proportions: Similar Triangles

Objective #3: The student will apply his knowledge of ratios and proportions to similar triangles by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

29. Define ratio and proportion.

30. Identify the properties of a proportion.

31. Know how to prove triangles similar.

32. Apply the relationships between segments of chords, secant segments, andtangent segments.

33. Apply the relationships between the altitude to the hypotenuse and the segments of the hypotenuse of a right triangle.

34. Apply the relationships among the legs and the hypotenuse of a right triangle.

35. Apply those theorems concerning lines parallel to one side of a triangle.

36. Apply those theorems concerned with bisecting an angle of a triangle.

37. Apply those ratios which concern corresponding sides of similar triangles.

38. Apply those theorems which concern ratios of chords and their segment, tangent segments and secant segments.

39. Understand and apply the Pythogorean Theorem to triangles, circles, and polygons.

40. Solve quadratic equations which may develop from these special ratios and theorems.

41. Find the lengths of the sides of a 30-60 right triangle.

IV. Areas of Polygons

Objective #4: The student will apply his knowledge of the concept of area to various polygons by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

42. Apply those theorems concerning areas of triangles, parallelograms, rectangles, and trapezoids.

43. Apply the Pythogorean Theorem to finding the areas of triangles and quadrilaterals.

44. Apply the special ratios of 30-60 right triangles to find the areas of triangles and similar polygons.

45. Apply those theorems concerning areas of similar triangles and similar polyms.

46. Find the area of a regular polygon.

47. Find the sum of interior and exterior angles of polygons.

48. Apply those theorems concerned with sides, perimeters, and areas of similar polygons.

49. Find the circumference and area of a circle.

50. Apply those ratios which concern radis, diameter, circumferences, and areas of circles.

51. Find the length of arc, area of a sector, and the area of a segment of a circle.

V. Inequalities and Locus

Objective #5: The student will apply his knowledge of the principles of inequalities and locus to problem solving by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

52. Apply those axioms and theorems concerning inequalities and inequalities of triangles.

53. Solve those problems in solving proofs of inequalities and computational problems.

54. Know the meaning Locus.

55. Determine the Locus of points concerning circles, parallel lines, perpendicular and perpendicular bisectors, angle bisectors, and triangles.

56. Know the meaning of intersection of two loci.

57. Determine the intersection of two loci.

58. Use the preparing for the college boards text to test comprehension of subject matter.

6. Construct the graph of the sine and cosine curve by using the unit circle and justify why both curves must be continuous and periodic.

7. Describe thebehavior of sine x and cosine x over given intervals.

8. Find the sin x, given the cos x where x is not a special real.

9. Derive the following reduction formulas:

   $\sin(\pi \pm x)$ $\cos(\pi \pm x)$

   $\sin(2\pi \pm x)$ $\cos(2\pi \pm x)$

   $\sin(-x)$ $\cos(-x)$

   by a. using the symmetries of the unit circle,

   b. graphing using equations of translations and solve related problems.

10. Derive $\sin(\frac{\pi}{2} - x) = \cos x$ and $\cos(\frac{\pi}{2} - x) = \sin x$ by using: In the same circle, equal chords have equal areas.

11. Derive the reduction formulas

    $\sin[(2n + 1) \pm x]$
    $\cos[(2n + 1) \pm x]$ where n ∈ nonegative integers

    by a. using symmetries to $\frac{\pi}{2} - x$ on unit circle,

    b. graphing using equations of translations.

    and solve related problems.

12. Derive the following formulas:

    $\cos(x_1 - x_2)$

    $\cos(x_1 + x_2)$

    $\sin(x_1 \pm x_2)$

    $\sin 2x$

    $\cos 2x$

    $\sin 1/2\ x$

    $\cos 1/2\ x$

and solve related problems.

13. Graph on real plane

$y = a \sin (bx + e) + d \Leftrightarrow y' = a \sin bx'$

$y = a \cos (bx + e) + d \Leftrightarrow y' = a \cos bx'$

by using translation of axis.

14. Write an equation of a sinusoidal curve given amplitude, period and phase shift.

15. Graph the sum of two sinusoidal curves by ordinate addition.

II. The Tangent, Cotangent, Secant, and Cosecant Functions

Objective #2: The student will apply his knowledge of all six trigonometric functions to derivations and problem solving by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

16. Define the tangent, cotangent, secant, and cosecant functions specifying domain and range.

17. Evaluate the circular functions of the special reals mentioned in #1e and #3.

18. Determine all circular functions from one given circular function.

19. Graph over the real plane $x \longrightarrow bf(x)$ where $f(x) \in \{\tan, \cot, \sec, \csc\}$

20. Derive the following reduction formulas:

$$f\ [(2n + 1)\frac{\pi}{2} + x]\ n \in I$$

$$f\ [n \cdot \frac{\pi}{2} = x]\ n \in \text{even } I$$

where $f = \{\tan, \cot, \sec, \csc\}$

by:

a. using definition of functions and symmetries of unit circle.

b. graphing using equations of translations and solve related problems.

21. Derive the following formula and solve related problems:

$\tan 2(x_1 \pm x_2)$

$\tan 2x$

$\tan \frac{1}{2} x$

22. Define the converse of each of the circular functions specifying the domain and range and graph each of the relations.

23. Explain why: a. it is necessary to: restrict the domain of circular functions for the existence of inverses.
b. the particular domains are selected.

24. Define and graph the inverses of the circular functions.

25. Derive the following formulas and apply them.

$\sin (\text{Sin}^{-1} u) = u$

$\cos (\text{Cos}^{-1} u) = u$

$\sin (\text{Cos}^{-1} u) = \sqrt{1-u^2}$

$\cos [\text{Sin}^{-1} u] = \sqrt{1-u^2}$

$\sin [\text{Tan}^{-1} u] = \frac{u}{\sqrt{1+u^2}}$

$\cos [\text{Tan}^{-1} u] = \frac{1}{\sqrt{1+u^2}}$

$\tan [\text{Sin}^{-1} u] = \frac{u}{\sqrt{1-u^2}}$

$\tan [\text{Cos}^{-1} u] = \frac{\sqrt{1-u^2}}{u}$

$\text{Cot}^{-1} u = \text{Tan}^{-1} (\frac{1}{u})$, where $u > 0$

$\text{Sec}^{-1} u = \text{Cos}^{-1} (\frac{1}{u})$, where $w \neq 0$

III. Angles and Arcs

Objective #3: The student will apply his knowledge of trigonometric functions to situations involving angles and arcs by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

26. Solve for a particular or a general solution of an open sentence that may require a substitution of any previously proven identity.

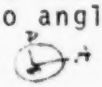
27. Derive and apply the sum and product formulas from the sum and difference formulas of the sine and cosine.

28. Distinguish between the geometric and trigonometric definition of an angle.

29. On the coordinate plane, recognize angles in standard position, coterminal angles, positive and negative angles.

30. Distinguish between the circular functions and the trigonometric functions.

31. Illustrate the correspondence from the coordinates of points $\underline{x}$ on the real number line to points $\underline{P}$ on the unit circle to angles $\angle AOP$ in standard position.



32. Illustrate why it is natural to define $\sin \angle AOP$ to be the ordinate of P and $\cos \angle AOP$ to be the abscissa of P.

33. Draw a diagram of angles in standard position in the unit circle and find the sine and cosine of the angles which correspond to the special reals.

34. Determine the sine and cosine of the angle in standard position given the coordinates of a point on the terminal side using similar triangles and the unit circle.

35. Prove that, if (a,b) are the coordinates of a point on the terminal side of an angle in standard position then

$$a^2 + b^2 \neq 0 \qquad \sin \angle AOP = \frac{b}{a^2 + b^2} \qquad \cos \angle AOP = \frac{a}{a^2 + b^2}$$

36. Determine the formulas for the remaining functions by using the results in #35.

37. Demonstrate the isomorphism between the Circular Functions and the Trigonometric Functions by showing that:

a. Definitions Correspond

Circular	Trigonometric
Real number on unit circle	Angle in standard position
Primary Real Number	Primary Angle
Domain set of reals	Domain set of angles

b. Operations are Preserved

38. Demonstrate how angle measure is related to circular arc measure by using the teal number line and the unit circle to find the unit are on circle C.

Unit are on C = $\frac{2}{h}$ where h = units contained in the circle.

39. Define the following units of angle measure: degree, radian, centangle.

40. Show that the relationship between w, the degree measure of an angle and y the radian measure of that angle is expressed by:

$$\frac{\pi}{180} = \frac{y}{w}$$

41. Use the proportion from #40 to convert from degree measure to radian measure and vice versa.

42. State the length of an arc intercepted by a given central angle and state the measure of the central angle intercepted by an arc with a given length.

43. Use a table of values of trigonometric functions to find:

a. the value of trigonometric functions

b. angles from a given numerical value of a trigonometric function of some angle.

44. Solve word problems that require construction of right triangles and computations with trigonometric functions.

45. Use the distance formula to derive the Law of Cosines and apply the Law of Cosines in solving problems.

46. Derive and find the area of a triangle given two sides and the included angle of a triangle.

47. Derive the Law of Sines and use this law to solve related problems.

48. Express complex numbers in polar or trigonometric form.

49. Perform the operations of addition, subtraction, multiplication, and division of complex numbers in polar form.

50. Solve equations using De Moivre's Theorem.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY HIS KNOWLEDGE OF ALGEBRAIC PRINCIPLES BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Mathematical Statements

Objective #1: The student will apply his knowledge of the principles of logic by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Distinguish between a sentence and a statement.
2. Define variable, replacement set, and open sentence.
3. Translate simple mathematical sentences using the universal and existential quantifiers.
4. Assign truth values to open sentences.
5. Write generalizations symbolically - using quantifiers, open sentences, and at most four variables - when given a set of statements.
6. Assign truth values to each of the following statements:

$\sim p$ $\quad$ $p \rightarrow q$

$p \vee q$ $\quad$ $p \leftrightarrow q$

$p \wedge q$

7. Assign truth values to composite statements using the basic truth tables given in #6 and at most four variables.
8. Negate simple, composite, and quantified statements such as:

$\sim (p)$ $\quad$ $\sim (p \rightarrow q)$

$\sim (p \vee q)$ $\quad$ $\sim [\forall, \exists \ (p)]$

$\sim (p \wedge q)$ $\quad$ $\sim [\exists, \forall \ (p)]$

9. Identify logically valid statements, logically equivalent statements and contradictory statements by constructing truth tables.

10. Identify valid statements by showing that the conclusion logically follows from the premises and recognize certain valid and invalid forms.

11. Write the conditional in the following forms:

    If p then q — q is a necessary condition for p

    p only if q — p is a sufficient condition for q

12. Write the variants of the conditional: the converse, inverse, and contrapositive.

valid: $\dfrac{p \vee q \quad \sim q}{p}$ $\dfrac{p \to q \quad p}{q}$ $\dfrac{p \to q \quad \sim q}{\sim p}$ invalid: $\dfrac{p \vee q \quad q}{p}$ $\dfrac{p \to q \quad q}{p}$ $\dfrac{p \to q \quad \sim p}{\sim q}$

13. Write the biconditional in the following forms:

    If p then q and if q then p

    p if and only if q

    p is a necessary and sufficient condition for q

14. Recognize the difference between proof by contradiction $(p \to q) \leftrightarrow \sim p \vee q$ and the indirect proof. $(\sim q \to \sim p)$

15. Illustrate by example how advertisers and propagandists make use of logical equivalence and invalid inference patterns.

16. Recognize examples of reflexive, symmetric, and transitive properties.

17. Define equivalence relation and recognize examples equivalence relations.

18. Use the elements of logic in mathematical proofs:

    $(x=y) \longrightarrow (x+z = y+z)$

    $(x=y) \longrightarrow (x-z = y-z)$

    $(x=y) \longrightarrow (xz = yz)$

    $[(z \neq 0) \wedge (x=y)] \longrightarrow \left(\frac{x}{z} = \frac{y}{z}\right)$

$$[(x=y) \wedge (z=w)] \rightarrow (x+z = y+w)$$

$$[(x=y) \wedge (z=w)] \rightarrow (xz = yw)$$

II. Real Number Properties

Objective #2: The student will increase his comprehension of the real number system by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

19. Define binary operation and recognize rules of combination defined over subsets of the reals which are binary operations.

20. Recognize examples of the following properties; and given an example over a subset of the reals, recognize whether the following properties hold for addition and multiplication:

    a. Closure

    b. Associative

    c. Existence of Unique Identity Elements

    d. Existence of Unique Inverse for Each Element

21. Demonstrate the properties of closure, associativity, identity, and inverse for modular arithmetic over multiplication and addition.

22. Demonstrate and describe the relation of "primeness" of module to the existence of a multiplicative and additive inverse.

23. Define a group and recognize whether a set defined over an operation is a group.

24. Recognize examples of commutative property and whether this property holds for the operations of multiplication and addition.

25. Define Abelian (Commutative) Group and recognize whether a set defined over an operation is an Abelian Group.

III. Proofs

Objective #3: The student will increase his ability to analyze given information to develop mathematical proofs by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

26. Write the proofs of the following statements:

$\forall x, y, z \in G,\ x = y \wedge z = w \rightarrow x * z = y * w$

$\forall x, y, z \in G,\ x * z = y * z \rightarrow x = y$

$\forall x \in G,\ (x^{-1})^{-1} = x$

27. Recognize examples of the distributive property of multiplication over addition and whether it holds for subsets of the reals.

28. Define field and recognize whether a set defined over the operation of addition and multiplication is a field.

29. Write the axiom, theorem, or definition justifying steps in proofs of properties of real numbers.

30. Write formal proofs of the following statements:

$(a = b) \rightarrow (a + e = b + c)$

$[(a = b) \wedge (c = d)] \rightarrow (a + e = b + d)$

$(b + e) = (-c) = b$

$(a + b = 0) \rightarrow (b = -a)$

$-(a + b) = (-a) + (-b)$

$a \cdot 0 = 0 \cdot a = 0$

$(-a)(b) = -ab$

$(x \neq b = a) \rightarrow (x = a + (-b))$

$(a = b) \rightarrow (as = be)$

$[(a = b) \wedge (e = d)] \rightarrow (ac = bd)$

$(be)\frac{1}{e} = b$ if $e \neq 0$

$\frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}$

$a(-1) = (-1)a = -a$

$(-a)(-b) = -ab$

$$xb = a \rightarrow x = a \cdot \frac{1}{b} \quad (b \neq 0)$$

31. Write formal proofs of theorems showing the relation between addition and subtraction, and the relation between multiplication and division.

32. Apply the properties of a field, theorems on addition, multiplication, subtraction, and division to evaluating expressions.

33. Write formal proofs of the following statements:

$[(x>y) \wedge (y\ z)] \rightarrow (x>z)$

$(x>y) \rightarrow [(x + z) > (y + z)]$

$[(x>y) \wedge (z \in R^+)] \rightarrow xz>yz$

$[(x>y) \wedge (z \in R^-)] \rightarrow xz<yz$

$[(x<y) \wedge (y<z)] \rightarrow (x\ z)$

$(x<y) \rightarrow [(x + z)<(y + z)]$

$[(x<y) \wedge (z \in R^+)] \rightarrow xz<yz$

$[(x<y) \wedge (z \in R^-)] \rightarrow xz>yz$

34. Establish the density of the real numbers by finding a real number between any two real numbers.

35. Define <u>completeness</u> and a <u>Complete Ordered Field</u>.

LEVEL OBJECTIVE

THE STUDENT WILL INCREASE IN HIS ABILITY TO ANALYZE THE PROPERTIES OF THE REAL NUMBER SYSTEM BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Rational Expressions

Objective #1: The student will increase his ability to analyze the rational number concepts to develop proofs by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Define and identify polynomials.

2. Perform basic operations of addition, subtraction, multiplication, and division of polynomials by using the laws of the field.

3. Use the Unique Factorization Theorem for Polynomials to completely factor polynomials which may be in the following forms:

   a. $zx + zy = z(x + y)$

   b. $x^2 + 2xy + y^2 = (x + y)^2$

   c. $x^2 - 2xy + y^2 = (x - y)^2$

   d. $x^2 - y^2 = (x + y)(x - y)$

   e. $acx^2 + (ad + bc) x + bd = (ax + b)(cx + d)$

   f. $ax + ay + bx + by = (a + b)(x + y)$

   g. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

   h. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

4. Use the Factor Theorem and the Remainder Theorem to factor polynomials not in any of the above forms.

5. Define and identify rational expressions.

6. Perform basic operations of addition, subtraction, multiplication, and division with rational exponents.

7. Define zero exponents and negative exponents as follows:

$\forall a \in$ Reals: If $a \neq 0$, $a^0 = 1$.

If $a \neq 0$, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^N$ where $n \in$ Reals$^+$

8. Define simplest form of rational expressions to mean no zero and no negative exponents.

9. Write formal proofs of: $\forall x, y \in$ Reals and $\forall m, n \in$ Integers

a. $x^m, x^n = x^{m+n}$

b. $(x^m)^n = x^{mn}$

c. $(xy)^m = x^m y^m$

d. $\frac{x^m}{x^n} = x^{m-n}$; $x \neq 0$

e. $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$, $y \neq 0$

10. Define fractional exponents as $a^{\frac{p}{q}} = \sqrt[q]{a^p}$

II. Irrational Expressions

Objective #2: The student will increase his ability to analyze the irrational number concepts to develop proofs by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

11. Define simplest form of Irrational expression to mean no fractional exponents and no fractions as radicands and no radicals in the denominator.

12. Write formal proofs of the following: $\forall p, m, n \in$ Integers and $\forall x, y \in$ Reals$^+$

a. $x^{\frac{m}{n}} = x^{\frac{mp}{np}} = \left(x^{mp}\right)^{\frac{1}{np}} = \left(x^{\frac{1}{np}}\right)^{mp}$

b. $(xy)^{\frac{1}{n}} = x^{\frac{1}{n}} y^{\frac{1}{n}}$ $\sqrt[n]{xy} = \sqrt[n]{x}\ \sqrt[n]{y}$

c. $x^{\frac{1}{m}}, x^{\frac{1}{n}} = x^{\frac{m+n}{mn}}$ $\sqrt[m]{x}\ \sqrt[n]{x} = \sqrt[mn]{x^{m+n}}$

d. $x^{\frac{1}{m}} \div x^{\frac{1}{n}} = x^{\frac{n-m}{mn}}$ $\sqrt[m]{x} \div \sqrt[n]{x} = \sqrt[mn]{x^{n-m}}$

e. $\left(x^{\frac{1}{n}}\right)^{\frac{1}{m}} = x^{\frac{1}{mn}}$ $\sqrt[n]{\sqrt[m]{x}} = \sqrt[mn]{x}$

13. Perform basic operations of addition, subtraction, multiplication, and division with Irrational expressions.

14. Use the properties of the Field to solve first degree equations in one variable.

15. Use properties of Ordered Field to solve inequalities in one variable.

16. Solve equations and inequalities in one variable involving absolute value.

17. Write formal proofs of the following:

a. $\forall x \in \text{Reals}\ |x| \geq x$ and $|x| \geq -x$

b. $\forall x, y \in \text{Reals}\ |x| + |y| \geq |x + y|$

c. $\forall x, y \in \text{Reals}\ |x \cdot y| = |x| \cdot |y|$

d. $\forall x, a \in \text{Reals}\ |x| < a \leftrightarrow -a < x < a$

e. $\forall x, a \in \text{Reals}\ |x| > a \leftrightarrow x < -a$ or $x > a$

LEVEL OBJECTIVE

THE STUDENT WILL APPLY HIS KNOWLEDGE OF THE PRINCIPLES OF THE REAL NUMBER SYSTEM BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Functions

Objective #1: The student will apply his knowledge of the principles of functions and equations to finding solution sets by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Define and write the Cartesian Product of two sets.
2. Define and identify relation, R, from one set to another set.
3. Define and identify the Domain and Range of a relation.
4. Define and identify function as Function, F, from set A to set B is a relation from A to B such that
    1. A is the domain of F
    2. If $(a,b) \in F$ and $(z,c) \in F$, then $b = c$.
5. Differentiate between into, onto, and one-to-one mappings.
6. Use function notation: $f = [(x,y)/y = f(x)]$ where $f(x)$ is the image of x.
7. Differentiate between an even and an odd function.
8. Relate even and odd functions to symmetry.
9. Define and identify periodic functions as $f(x) = f(x + h)$.
10. Perform the addition, subtraction, multiplication, and division of functions.
11. Define and form the composition of functions.

12. Form the converse of a function and indicate whether the converse is a function called the inverse.

13. Indicate whether a function is increasing, decreasing, constant, or neither over a particular interval.

14. Define linear function.

15. Define and write the inverse of a linear function.

16. Define and find the zero of a linear function.

17. Graph linear functions using slope, intercepts, and other points.

18. Graph and write the equation of linear functions parallel to the given function.

19. Graph and write the equation of a linear function perpendicular to the given function.

20. Define quadratic function and differentiate between linear and quadratic functions.

21. Find the solution set of the quadratic function, $f(x) = 0$, by factoring, completing the square, and the quadratic formula.

22. Find the discriminant to identify the type of solution of a quadratic equation.

23. Relate the sum and product of the roots to the coefficients of a quadratic function.

24. Write the equation of a quadratic function using the sum and product of the roots.

25. Solve quadratic inequalities.

26. Solve irrational equations, checking for extraneous roots.

II. Conic Sections

Objective #2: The student will increase his comprehension of conic sections by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

27. Define circle.

28. Define parabola.

29. Define ellipse.

30. Define hyperbola.

31. Derive the equation for the above conic sections at the origin and translated to the point (h,k).

32. Recognize the properties as they relate to each conic:

    a. vertex

    b. axis of symmetry

    c. focus

    d. latus rectum

    e. eccentricity

    f. directrix

III. Logarithms

Objective #3: The student will apply his knowledge of logarithms to mathematical operations by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

33. Define the exponential function.

34. Graph the exponential function.

35. Define the logarithmic function as the inverse of the exponential function.

36. Differentiate between common logarithms and logarithms with other bases.

37. Use the properties of logarithms to perform multiplication, division, powers, and roots.

38. Use linear interpolation to gain a greater degree of accuracy.

IV. Complex Numbers

Objective #4: The student will apply his knowledge of the principles of a numeration system to include complex numbers by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

39. Define complex numbers as a + bi and (a,b).

40. Recognize and prove the Field Properties for the set of complex numbers:

    a. Identify of addition and multiplication

    b. Inverses for addition and multiplication

    c. Closure of addition and multiplication

    d. Associativity of addition and multiplication

    e. Commutativity for addition and multiplication

    f. Distributive of multiplication over addition

41. Perform the basic operations of addition, multiplication, subtraction, and division with complex numbers.

42. Simplify complex numbers using the Field Properties and the conjugate.

43. Graph a complex number.

44. Perform the operations of addition and subtraction graphically.

V. Polynomials and Sequences

Objective #5: The student will apply his knowledge of mathematical reasoning to polynomials and number sequences by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

45. Define polynomial function.

46. Define rational function.

47. Use synthetic division to shorten division problems.

48. Utilize the Factor Theorem, Remainder Theorem, and Synthetic Division to factorhigher degree polynomials.

49. Utilize descartes Rule of Signs to solve higher degree polynomial functions.

50. Find the Complex zero of real polynomials.

51. Graph polynomial functions.

52. Use Mathematical Induction as a method of proof.

53. Define and recognize Arithmetic sequences.

54. Define first term, last term, nth term, and common difference.

55. Derive and the formula for finding the first term, nth term, number of terms, or the common difference for an Arithmetic Sequence.

56. Define and find arithmetic means between any two terms of a sequence.

57. Define and derive a formula for the sum of an Arithmetic Series.

58. Define and recognize Geoemtric Sequences.

59. Define first term, last term, nth term, and common factor.

60. Derive and use the formula for finding the first term, nth term, number of terms, or common factor.

61. Define and find Geometric means between any two terms of a sequence.

62. Define and derive the formulas for the sum of a finite and infinite Geometric Series.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY HIS KNOWLEDGE OF TRIGONOMETRY TO PROOFS AND PROBLEMS BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Functions

Objective #1: The student will apply his knowledge of trigonometric functions by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Define a function as a set of ordered pairs no two of which have the same first element.
2. Recognize a function from both its equation and its graph.
3. Differentiate among an "into" function, an "onto" function, and a "one-to-one" function.
4. Define a radian.
5. Find the radian measure of a central angle of a circle given its arc length and the radius of the circle.
6. Convert from radian measure to degree measure and vice versa.
7. Understand that the wrapping function (O) maps the real numbers onto the points of a unit circle.
8. Find the points onto which certain real numbers, such as 0, $\pi/3$, $\pi/4$, $\pi/6$, $\pi/2$, π, $3\pi/2$, etc. are mapped.
9. Recognize the basic definitions of $\sin\theta$ and $\cos\theta$.
10. Find $\sin\theta$ and $\cos\theta$ for real number values of θ in all four quadrants.
11. Identify the definitions of $\tan\theta$, $\operatorname{ctn}\theta$, $\sec\theta$, and $\csc\theta$ in terms of $\sin\theta$ and $\cos\theta$.
12. Apply the basic relationships that exist among the six circular functions.

13. Recognize that the six circular functions are periodic in nature.

14. Describe the behavior of the circular functions over given intervals.

15. Graph the six circular functions with the following modifications:

    a. Change of amplitude,

    b. Change of period,

    c. Horizontal translation,

    d. Vertical translation,

    e. Addition or subtraction of curves, and

    f. Combinations of the above.

16. Apply the eight fundamental relationships that exist among the circular functions in proving identities.

17. Develop techniques which may be employed in the proving of identities.

18. Apply the various sets of formulas which may be developed from the basic definitions:

    a. Addition formulas,

    b. Subtraction formulas,

    c. Double-angle formulas,

    d. Half-angle formulas, and

    e. Sum and difference formulas.

19. Identify the inverse of a function, in particular, the inverse circular functions.

20. Graph the inverse circular functions and recognize their ranges of principal values.

21. Evaluate expressions using the principal values of the inverse circular functions.

22. Solve open sentences involving the circular functions.

II. Complex Numbers and Triangles

Objective #2: The student will apply his knowledge of the principles of real numbers and trigonometry to problems involving complex numbers and triangles by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

23. Express complex numbers in polar or trigonometric form.

24. Perform the operations of addition, subtraction, multiplication, and division of complex numbers in polar form.

25. Solve equations using DeMoivre's Theorem.

26. Redefine the circular functions as trigonometric in terms of the sides of a triangle.

27. Solve for the various parts of a right triangle using traditional methods and logarithms.

28. Solve for the various parts of an oblique triangle using the Law of Sines or the Law of Cosines in the following situations:

    a. AAS,

    b. SSA (Ambiguous Case),

    c. SAS, and

    d. SSS.

LEVEL OBJECTIVE

THE STUDENT WILL INCREASE IN HIS ABILITY TO ANALYZE THE PRINCIPLES OF GEOMETRY BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Points and Lines

Objective #1: The student will increase his ability to analyze the properties of points and lines by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Set up a coordinate system on a line.
2. Set up a coordinate system on a plane.
3. Compute the distance between two points on a line.
4. Graph the solution set for the following:
   a. $x \geq a \quad (a \in R)$,
   b. $x \leq a \quad (a \in R)$,
   c. $|x| = a \quad (a \in +R)$,
   d. $|x| \geq a \quad (a \in +R)$,
   e. $|x| \leq a \quad (a \in +R)$, and
   f. $a \leq |x| \leq b \quad (a < b, a, b \in +R)$.
5. Compute the distance between two points on a plane.
6. Apply the sistance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to various situations which may arise in analytic geometry.
7. Compute the coordinates of a point which bisects a given segment.
8. Determine the coordinates of the points which divide a given segment into n congruent segments.

9. Identify the slope of a line as the tangent of its angle of inclination.

10. Compute the slope of a line given two points through which it passes.

11. Recognize the various forms of the equation of a straight line:

    a. Two-point form,

    b. Point-slope form,

    c. Slope-intercept form,

    d. Intercept form, and

    e. Standard form.

12. Write the equation of a straight line in the above five forms given appropriate information.

13. Apply the fact that the slopes of parallel lines are equal to problem situations.

14. Apply the fact that the slopes of perpendicular lines are the negative reciprocals of each other to problem situations.

15. Find the angle of intersection between two lines.

16. Determine theangles of a polygon knowing the coordinates of its vertices.

17. Compute the distance from a point to a line.

18. Apply the formula for the distance from a point to a line to various situations which arise in analytic geometry.

19. Apply the methods of coordinate geometry to the proofs of theorems.

20. Apply the methods of coordinate geometry to finding certain loci of points.

II. Conic Sections

Objective #2: The student will increase his ability to analyze conic sections by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

21. Recognize that a circle is the locus of points in a plane at a given distance from a given point in the plane.

22. Determine the center and radius of a circle when its equation is written in the following forms:

    a. $x^2 + y^2 = r^2$, and

    b. $(x - h)^2 + (y - k)^2 = r^2$.

23. Write the equation of a circle given various sets of information.

24. Define the eccentricity of a conic as a positive number which represents the ratio of the distance from a point on the conic to a given focal point to the distance from the same point on the conic to a given directrix line.

25. Recognize that the parabola is the conic section with eccentricity e = 1.

26. Graph a parabola when its equation is written in one of the following forms:

    a. $x^2 = 4py$,

    b. $y^2 = 4px$,

    c. $(x - h)^2 = 4p(y - k)$, and

    d. $(y - k)^2 = 4p(x - h)$.

27. Recognize that the ellipse is the conic section with e 1.

28. Graph an ellipse when its equation is written in one of the following forms:

    a. $\frac{x}{a^2} + \frac{y}{b^2} = 1$,

    b. $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$ $\quad a > b$,

    c. $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$, and

d. $\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1.$

29. Recognize that the hyperbola is the conic section with e 1.

30. Graph a hyperbola when its equation is written in one of the following forms:

a. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$

b. $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1,$

c. $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1,$ and

d. $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1.$

31. Write the equations of the parabola, ellipse, and hyperbola given certain sets of information.

32. Find the coordinates of the points of intersection of various pairs of conics.

33. Recognize the equations of degenerate conics and graph these special cases.

34. Recognize that the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is the general equation of the second-degree.

35. Eleminate the cross-product (xy) term from the above equation by a rotation of axes.

36. Graph the transformed conic on the new set of rotated axes.

III. Parameters and Coordinates

Objective #3: The student will increase his ability to analyze equations and the possible coordinate systems of graphing by completing the following suggested activities to the satisfaction of the teacher.

Activities:

37. Recognize that a parameter is a third variable which relates the independent and dependent variables, x and y.

38. Graph a curve from its parametric equations.

39. Eliminate the parameter and graph the resulting Cartesian equation.

40. Determine the equivalency of the parametric and Cartesian equations.

41. Construct a polar coordinate system.

42. Plot the points (r, 0) on a polar coordinate system.

43. Interchange the polar and Cartesian coordinates of points.

44. Determine the symmetries of an equation when written in polar form.

45. Graph curves in polar from such as the following:

    a. Cardioid,

    b. Limacon,

    c. Lemniscate,

    d. Rose Curves,

    e. Spiral, and

    f. Lituus.

46. Transform an equation from Cartesian to polar form and vice versa.

47. Realize that some surves are graphed more easily in Cartesian form and others more easily in polar form.

48. Compute the distance between two points when their coordinates are given in polar form.

49. Determine the symmetries of a curve when its equation is written in Cartesian form.

50. Find the equations of the horizontal and vertical asymptotes of rational functions.

51. Graph rational functions with their appropriate horizontal and vertical asymptotes.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY HIS KNOWLEDGE OF CALCULUS TO PROBLEM SOLVING SIUTATIONS BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Functions

Objective #1: The student will increase his comprehension of functions by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Recognize a polynomial function as one which has an equation $y = P(x) = a_0x^n + a_1x^{n-1} + \ldots + a_n$ where n is a positive integer and $a_0, a_1, \ldots a_n$ are real numbers.

2. Identify polynomial functions as constant, linear, quadratic, cubic, biquadratic, etc.

3. Find the domain and range of a polynomial function.

4. Understand the intuitive definition of a continuous function.

5. Compute the average rate of change of a function.

6. Recognize the average rate of change of a function as the slope of a secant line to the curve.

7. Apply the concept of the average rate of change of a function to practical situations.

8. Recognize the exact rate of change of a function as the limit of the average rate of change as $\Delta x > 0$.

II. Derivatives

Objective #2: The student will apply his knowledge of derivatives in calculus to the solution of problems by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

9. Identify the derivative of a function as its exact rate of change.

10. Identify the exact rate of change or the first derivative of the function as the slope of a tangent line to the curve.

11. Compute the first derivative of polynomial functions.

12. Write the equations of tangent lines to a curve at any point on the curve.

13. Recognize the average velocity as the average rate of change of the distance function.

14. Recognize the instantaneous velocity as the first derivative of the distance function.

15. Recognize the average acceleration as the average rate of change of the velocity function.

16. Recognize the instantnaeous acceleration as the first derivative of the velocity function or the second derivative of the distance function.

17. Apply the theory of distance, velocity, and acceleration to practical problems.

18. Write the equations of normal lines to a curve at any point on the curve.

19. Differentiate between an increasing and a decreasing function.

20. Recognize points of relative maximum or minimum on a curve.

21. Find the critical values for a polynomial function in x.

22. Find the maximum and minimum points on a curve by using the first derivative test.

23. Find the maximum and minimum points on a curve by using the second derivative test.

24. Apply maxima and minima theory to the graphing of polynomial functions.

25. Apply maxima and minima theory to the solution of practical problems.

III. Integrals

Objective #3: The student will apply his knowledge of the concepts of integration to solving problems by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

26. Determine the antiderivative or integral of a polynomial function.

27. Apply integration to distance, velocity, and acceleration problems.

28. Evaluate a definite integral.

29. Apply integration to the computation of areas.

LEVEL OBJECTIVE

THE STUDENT WILL APPLY HIS KNOWLEDGE OF THE CONCEPTS OF PROBABILITY AND MATHEMATICAL INDUCTION TO PROBLEMS BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Permutations and Combinations

Objective #1: The student will apply his knowledge of the concepts of permutations and combinations to problems by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Apply the fundamental principle of counting in problem situations.
2. Differnetiate between an "and" and an "or" connective when using the fundamental principle of counting.
3. Realize that a permutation is the arrangement of a group of objects in a definite order.
4. Use the following formulas in problem situations:
   a. $n^Pn = n!$,
   b. $n^Pr = \frac{n!}{(n - r)!}$, and
   c. Repeated objects - $\frac{n!}{p!q!.....}$.
5. Realize that a combination or a selection is a group of things in which the arrangement or order is not considered.
6. Use the formula $n^Cr = \frac{n!}{r!\ (n - r)!}$ in problem situations.
7. Differentiate between permutation and combination examples.

II. Probability

Objective #2: The student will apply his knowledge of the concept of probability to situations involving mathematical expectation by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

8. Realize that a probability is the ratio of the number of ways an event can occur successfully to the total number of ways the event can occur.

9. Identify events that are mutually exclusive.

10. Compute the probability of mutually exclusive events.

11. Compute the probability of independent events.

III. Mathematical Induction

Objective #3: The student will apply his knowledge of inductive reasoning to proofs by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

12. Differentiate between the methods of inductive and deductive thinking.

13. Define the concept of mathematical induction.

14. Realize that the two requirements for mathematical induction are sufficient to prove the validity of sentences.

15. Test the validity of sentences by the method of mathematical induction.

LEVEL OBJECTIVE

THE STUDENT WILL DEMONSTRATE AN INCREASE IN HIS ABILITY TO ANALYZE THE REAL NUMBER SYSTEM AND ALGEBRAIC EXPRESSIONS TO DEVELOP THE MATHEMATICS OF CALCULUS BY COMPLETING THE FOLLOWING LEVEL TO THE SATISFACTION OF THE TEACHER.

I. Analytic Preparation

Objective #1: The student will increase his ability to analyze geometric concepts by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

1. Review those topics of Analytic Geometry covered in the Advanced Mathematics course.
2. Consider all definitions of the absolute value of x.
3. Consider geometric interpretations of x.
4. Analyze the triangle inequality and the absolute value of sums, products, quotients, and differences.
5. Analyze the four kinds of intervals of real numbers.
6. Examine the definition for neighborhoods.
7. Analyze and apply the definition for a deleted neiborhood.

II. Functions

Objective #2: The student will apply his knowledge of the concepts of functions by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

8. Investigate and use the definitions of a function.
9. Determine when two functions are equal.

10. Use the different methods of representing the functional relationship between the variables x and y which are:

    a. by tables of values.

    b. by corresponding scales.

    c. by means of graphs.

    d. by means of equations.

11. Recognize and graph the greatest integer function.

12. Recognize and graph the signium function.

13. Evaluate the composition of functions.

14. Investigate and study the behavior of functions.

15. Compute and graph the slope function.

16. Formulate the method for finding the derivative of a function.

17. Investigate and solve problems regarding velocity and rates.

18. Understand the definition of limit as an operation applied to a function at a point.

19. Prove theorems about limits of sums, products, and quotients of functions.

20. Examine and apply the properties of infinity.

III. Derivatives of Algebraic Functions

Objective #3: The student will apply his knowledge of the derivative of a function by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

21. Identify a polynomial in x as a function which is the sum of a finite number of monomial terms.

22. Prove that the derivative, with respect to x, of x^n is nx^{n-1} when n is any positive integer.

23. Prove that the derivative of a constant is zero.

24. Prove that if $u=f(x)$ is a differentiable function of x and c is a constant, then $\frac{d\,(cu)}{dx} = c\,\frac{du}{dx}$.

25. Prove that the derivative of the sum of a finite number of differentiable functions is equal to the sum of their derivatives.

26. Apply the use of derivatives to mechanics by using the first derivative to find the velocity of a moving body and by using the second derivative to find acceleration.

27. Derive formulas for finding the derivative with respect to x of products ($y=uv$), quotients ($y=\frac{u}{v}$), and powers ($y=u^n$).

28. Calculate dy/dx from a method known as implicit differentiation.

29. Estimate the change of y produced in a function $y=f(x)$ when x changes by a small amount x.

30. Derive and use the chain rule for derivatives.

31. Adopt the definitions of dx and dy in differential notation.

32. Use the geometric interpretation of dy and dx.

33. Use differentials to obtain reasonable approximations.

34. Recognize the continuity of a function.

35. Consider how the sign of the derivative at a point gives information about the curve.

36. Solve problems involving related rates.

37. Examine the significance of the sign of the second derivative.

38. Follow the procedure for graphing the equation $y=f(x)$.

39. Analyze the theory regarding maxima and minima.

40. Solve problems involving maxima and minima.

41. Derive and apply Rolle's Theorem.

42. Prove and apply the Mean Value Theorem.

IV. Integration

Objective #4: The student will apply his knowledge of the concept of integration by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

43. Analyze the meaning of integration.

44. Solve differential equations.

45. Find a function by use of the indefinite integral.

46. Consider the applications of indefinite integration.

47. Review formulas and theory from trigonometry.

48. Prove the theorem $\frac{\lim}{2-0} \frac{\sin 2}{2} = 1$.

49. Derive the derivatives and integerals of y=sin u and y=cos u.

50. Find the area under a curve.

51. Compute areas as limits.

52. Find areas by calculus.

53. Derive and apply the definite integral and the Fundamental Theorem of Integral Calculus.

54. Approximate an integral by using the trapexoidal rule.

55. Use summation notation.

56. Find the area between two curves.

57. Calculate the distance traveled by a body moving with velocity v=f(t).

58. Solve problems regarding the volume of a solid of revolution by the disc method and by cylindrical shells.

59. Find the length of a plane curve.

60. Solve problems regarding the area of a surface of revolution.

61. Find the average value of a function.

62. Solve problems regarding centroid and center of gravity.

63. Solve problems regarding moments and center of mass.

64. Solve work problems.

V. Transcendental Functions

Objective #5: The student will increase his ability to analyze trancendental functions by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

65. Study the meaning of the word transcendental.

66. Derive the other trigonometric functions not previously derived.

67. Examine and review the inverse trigonometric functions.

68. Analyze thenatural logarithm of x.

69. Derive the derivative of ln x.

70. Establish the properties of the natural logarithms.

71. Graph y=ln x.

72. Analyze the exponential function.

73. Analyze the function a^u.

74. Establish the properties for the function a^u.

75. Analyze the function $\log_a u$.

VI. Mehtods of Integration

Objective #6: The student will increase his ability to analyze various methods of integration by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

76. Review the basic formulas.

77. Learn and apply the formula for powers of trigonometric functions.

78. Examine and apply the method used for even powers of sines and cosines.

79. Analyze and apply the method used for integrals involving the square root of a^2-u^2, square root of a^2+u^2, square root of u^2-a^2, a^2+u^2, and a^2-u^2.

80. Analyze and apply the method used for integrals involving ax^2+bx+c.

81. Integrate by the method of partial fractions.

82. Integrate by parts.

83. Integrate rational functions of sin x and cos x and other trigonometric integrals.

84. Integrate by using further substitutions.

85. Investigate improper integrals and their convergence.

86. Use Simpson's Rule for approximation.

VII. Plane Analytic Geometry

Objective #7: The student will increase his ability to analyze geometry by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

87. Review those topics of Analytic Geometry covered from the Advanced Mathematics course.

88. Graph higher degree equations using horizontal and vertical asymptotes.

89. Find equations of the tangent and normal lines.

90. Review those topics of Polar Coordinates covered from the Advanced Mathematics course.

91. Review the graphs of polar equations.

VIII. Hyperbolic Functions

Objective #8: The student will increase his ability to analyze hyperbolic functions by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

92. Analyze the meaning of hyperbolic functions.

93. Compare hyperbolic functions with circular functions.

94. Derive the equations of the hyperbolic sine and cosine.

95. Derive the remaining hyperbolic functions in terms of the sinh u and cosh u.

96. Graph the hyperbolic functions.

97. Establish the derivatives and integrals for the hyperbolic functions.

98. Establish the geometric significance of the hyperbolic radian.

99. Analyze the inverse hyperbolic functions.

100. Establish the derivatives and integrals for the inverse hyperbolic functions.

101. Derive the equation of the hanging cable.

102. Define and sketch the curve referred to as the gudermaninan of x.

IX. Vectors and Parametric Equations

Objective #9: The student will increase his ability to analyze vectors and parametric equations by completing the following suggested activities to the satisfaction of the teacher.

Activities:

The student will be able to:

103. Analyze parametric equations in kinematics.

104. Examine the use of parametric equations in analytic geometry.

105. Analyze the use of vector components and the unit vectors i and j.

106. Differentiate vectors.

107. Examine a unit vector tangent to the curve at P.

108. Explore curvature and normal vectors.

109. Examine tangential and normal components of the velocity and acceleration vectors.

110. Investigate the particle P moving on a curve whose equation is given in polar coordinates.